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Sensor Scheduling for Energy Constrained Estimation in Multi-Hop Wireless Sensor Networks

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Abstract— **Wireless Sensor Networks (WSNs) enable a wealth of new applications where remote estimation is essential. Individual sensors simultaneously sense a dynamic process and transmit measured information over a shared channel to a central fusion center. The fusion center computes an estimate of the process state by means of a Kalman filter. In this paper we assume that the WSN admits a tree topology with fusion center at the root node. At each time step only a subset of sensors can be selected to transmit their observations to the fusion center due to limited energy budget. We propose a stochastic sensor selection algorithm to randomly select a subset of sensors according to certain probability distribution, which is chosen to minimize the expected next step estimation error covariance matrix while maintaining the connectivity of the network. One of the main advantages of the stochastic formulation over the traditional deterministic formulation is that the stochastic formulation provides smaller expected estimation error than the deterministic formulation. Further, we prove that the optimal stochastic sensor selection problem can be relaxed into a convex optimization problem and thus solved efficiently. We also provide a possible implementation of our algorithm which does not introduce any communication overhead. Finally a numerical example is provided to show the effectiveness of the proposed approach.**

I. Introduction

Sensor networks span a wide range of applications, including environmental monitoring and control, health care, home and office automation and traffic control [1]. In these applications, estimation algorithms like Kalman filters can be used to perform state estimation based on lumped-parameter models of the physical phenomena. However, WSN operating constraints such as power often make it difficult to collect data from every sensor at the sampling rate required for effective monitoring. These considerations have led to the development of sensor scheduling strategies to select, at each time step, the subset of reporting sensors that minimizes a certain cost function, usually related to the expected estimation error.

Sensor network energy consumption minimization and consequently lifetime maximization problems have been active areas of research over the past few years, as researchers realized that energy limitations constitute one of the major obstacles to the adoption of such technology. Sensor network energy minimization is typically done via efficient MAC protocol design [2], or via efficient scheduling of the sensor states [3], [4]. In [5] Xue and Ganz show that the lifetime of the sensor networks is influenced by transmission schemes, network density and transceiver parameters with different constraints on network mobility, position awareness and maximum transmission range. Chamam and Pierre [6] propose a sensor scheduling scheme to optimally put sensors in active or inactive modes. Shi et. al [7] consider sensor energy minimization as a mean to maximize network lifetime while guaranteeing a desired quality of estimation accuracy. Moreover in [8] they propose a sensor tree scheduling algorithm which leads to longer network lifetime. In all the above papers, the authors consider the estimation performance in terms of the steady state value of the error covariance matrix given by the Kalman filter.

Another important contribution is the work of Joshi and Boyd [9] where the general single step sensor selection problem is formulated and solved by means of convex relaxation techniques. Such a paper provides a very general framework that can handle various performance criteria and energy and topology constraints. Following this work, Mo et al. [10], [11] show that multi-step sensor selection problem can also be relaxed into a convex optimization problem and thus can be solved efficiently.

A very different approach w.r.t. the above deterministic solutions has been proposed in [12]. There a stochastic sensor selection algorithm is proposed based on the idea that at each step the sensors randomly choose to send measurements or not according to a certain probability distribution. In this view probability distributions become the optimization parameters chosen so as to minimize the expected estimation error covariance at steady state. The authors argue that such a stochastic approach has several advantages over the conventional approaches. For example, it is easier, in this framework, to take into account random communication channel failure, which is common in wireless sensor networks. A relevant limitation of the result presented in that paper is rooted in the assumptions that the sensor network has a star topology and that only one sensor can transmit during one sampling period, which requires precise coordination between sensors.

In the present work, we propose a stochastic sensor selection algorithm that not only overcomes the above limitation but also solves the routing problem for the case the wireless

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sensor network has a tree topology.

At each sampling period, the sensors are randomly selected to minimize the expected next step estimation error covariance while maintaining the connectivity of the network. In order to make the optimization problem tractable the problem is relaxed by only considering the minimization of suitable approximation of the expected estimation error. Such a choice reduces the optimal sensor selection schedule to a convex problems. An efficient technological expedient that allows implementation of the proposed sensor selection strategy without any communication overhead will be also discussed. The rest of the paper is organized as follows. In Section II we describe the sensor selection problem for tree topology networks. In Section III, we propose a stochastic sensor selection algorithm and compute the best schedule which minimizes the lower-bound of the expected next step estimation error covariance matrix. Some implementation aspects will be discussed at the end of the section. A numerical example regarding the sensor network monitoring of a diffusion process is provided in Section IV while Section V concludes the paper.

II. Problem Formulation

Consider the following LTI system whose state dynamics are given by

$$x_{k+1} = Ax_k + w_k \tag{1}$$

where $x_k \in \mathbb{R}^n$ represents the state, w_k, x_0 are independent Gaussian random vectors. $x_0 \sim \mathcal{N}(0,\ \Sigma)$ and $w_k \sim \mathcal{N}(0,\ Q)$, where Σ, $Q \geq 0$ are positive semidefinite matrices.

A wireless sensor network composed of m sensing devices $s_1, \ldots, s_m$ and one fusion center s_0 used to monitor the state of system (1). The measurement equation is

$$y_k = Cx_k + v_k, \tag{2}$$

where $y_k = [y_{k,1}, y_{k,2}, \ldots, y_{k,m}]' \in \mathbb{R}^m$ is the measurement vector[1]. Each element $y_{k,i}$ represents the measurement of sensor i at time k. $C = [C_1', \ldots, C_m']'$ is the observation matrix. $v_k \sim \mathcal{N}(0,\ R)$ is the measurement noise, assumed to be independent of x_0 and w_k. We also assume that $R = diag(r_1, \ldots, r_m)$ is diagonal, which means that the measurement noise at each sensor is independent of each other, and each $r_i > 0$.

To model the communication among nodes let us introduce an oriented communication graph $G = \{V\,, E\}$, where the vertex set $V = \{s_0, s_1, \ldots, s_m\}$ contains all sensor nodes including the fusion center. The set of edges $E \subseteq V \times V$ represents the available connections between nodes, i.e. $(i, j) \in E$ implies that the node s_i may send information to the node s_j. Each node of the sensor network acts as a gateway i.e. every time it communicates to another node it sends, in a single packet, its own measurements together with all the data received from the other nodes.

In this paper we assume that, for every sensor in the sensor network, there exists one and only one path to the fusion center, i.e. the sensor network has a directed tree topology. Moreover we assume each link to have an associated weight $c(e_{i,j})$ that indicates the energy consumed if s_i directly transmits a packet to s_j. Hoping to increase legibility, we will sometimes abbreviate $c(e_{i,j})$ as c_i, $i = 1, \ldots, m$, since, in the assumed topology, each sensor node has only one outgoing edge.

Since sensor measurements usually contain redundant information, in order to reduce the energy cost, it is desirable to only use a subset of sensors at each sampling time. Moreover, in a tree topology, we cannot select arbitrary subsets of nodes but we are forced to select nodes (and connections) such that for each sensor node there exists an enabled path to the fusion node. As a result, we define a transmission topology of G as a possible subtree $T = \{V_T\,, E_T\}$ of G where the fusion center $s_0 \in V_T$, $V_T \subseteq V$, $E_T \subseteq E$. V_T denotes the selected subset of sensors and E_T denotes the communication links used by sensors to transmit observations back to fusion center. We also define the set of all possible transmission topologies T to be $\mathcal{T}$, i.e. the set of all possible subtrees of G containing s_0.

It is straightforward to see that, for a transmission tree T, the total energy cost, for the transmission over the network is[2]

$$\mathcal{E}(T) = \sum_{e \in E_T} c(e).$$

Suppose now that at time k, a particular transmission tree T is used with $V_T = \{s_0,\ s_{i_1}, \ldots, s_{i_j}\}$ and let us define two matrices C_T and R_T to be

$$C_T \triangleq [C_{i_1}',\ C_{i_2}', \ldots, C_{i_j}']',\ R_T \triangleq diag(r_{i_1}, \ldots, r_{i_j}). \tag{3}$$

It is easy to prove that the Kalman filter with a time varying observation matrix C_T is still the optimal filter in the case of sensor selection. Hence, once the fusion center collects all the observations at time k, it will use the Kalman filter to compute the optimal estimate of current state $\hat{x}_k$ and the estimation error covariance matrix P_k, which satisfy the following equations

$$\hat{x}_k = K_k(y_k - C_T\hat{x}_{k|k-1}) + \hat{x}_{k|k-1}, \tag{4}$$

$$P_k = P_{k|k-1} - K_k C_T P_{k|k-1}, \tag{5}$$

where

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1}, P_{k|k-1} = AP_{k-1}A' + Q, \tag{6}$$

$$K_k = P_{k|k-1}C_T'(C_T P_{k|k-1} C_T' + R_T)^{-1}. \tag{7}$$

Let us introduce also the information filter, to be used in the next section. Define the information matrix Z_k at time k to be

$$Z_k \triangleq P_k^{-1}. \tag{8}$$

Z_k satisfies the following recursive equations

$$Z_k = Z_{k|k-1} + C_T' R_T^{-1} C_T, \tag{9}$$

[1]The $'$ on a matrix always means transpose.

[2]Here we have to assume that $cost(e_{i,j})$ is constant regardless of number of observations contained in the packet. This is realistic in many cases. In fact, if measurements are of simple type (e.g. low precision scalar values), then the transmission overhead (e.g. header, hand shaking protocol) dominates the payload.

where

$$Z_{k|k-1} = (AZ_{k-1}^{-1}A' + Q)^{-1}. \tag{10}$$

Since we already assumed that R is diagonal, it is easy to see that (9) can be written as

$$Z_k = Z_{k|k-1} + \sum_{s_i \in V_T,\, s_i \neq s_0} \frac{C_i C_i'}{r_i}. \tag{11}$$

The optimal sensor selection problem under investigation can be formulated as follows:

Problem 1 (Sensor Selection over Transmission Tree): Determine the transmission tree that minimizes the trace of the estimation error covariance matrix P_k such that the overall transmission cost is less of the maximum energy budget $\mathcal{E}_d$, i.e.

$$\min_{T \in \mathcal{T}} \; trace(P_k)$$
$$\mathcal{E}(T) \leq \mathcal{E}_d,$$

Remark 1: Solving Problem 1 with an exhaustive search over the space $\mathcal{T}$ is a very hard problem even for a small sensor network. To give an idea of the size of the problem, consider the case where G is a perfect binary tree of 63 nodes: the number of candidate solutions, i.e. all possible transmission graphs T, is of the order of 10^{11}.

Remark 2: In Problem 1 we require that at each sampling time the total energy cost does not exceed a certain energy budget. In real applications different constraints may appears (e.g. requirements on the sensor lifetime). However, it can be shown (see e.g. [9]) that many of these constraints are linear or convex inequalities and therefore can be easily integrated in any convex optimization problems.

To address the impracticality of exhaustive searches, we propose a stochastic selection strategy to be discussed in the next section. We will see how a stochastic formulation may be more desirable.

III. Main Result

The main idea behind the proposed method is to optimally design the probability that a certain transmission tree T is selected at time k, denoted by $\pi_{k,T}$. The main insight behind this approach is to substitute binary variables in Problem 1 with continuous variables and relaxing the problem. The proposed approach can then be summarized as follows: first we determine the probabilities $\pi_{k,T}, \forall k > 0, \forall T \in \mathcal{T}$ that minimize an objective function related to the expected estimation error covariance $\mathbb{E}P_k$; then at each time k we randomly select a transmission tree $T \in \mathcal{T}$ on the basis of the probabilities $\pi_{k,T}, \forall T \in \mathcal{T}$.

The remainder of the section is organized as follows. In the first subsection we will introduce the notation and the main features of our algorithm. Then, in the second subsection, we will show how to compute upper and lower bound of $\mathbb{E}P_k$ and thus relax the random transmission tree selection to a convex optimization problem. Finally in the last subsection, we will discuss how to implement the proposed sensor selection strategy so as to avoid communication overhead.

A. Description of Random Transmission Tree Selection Algorithm

Suppose that at each time k, we randomly select a tree T from $\mathcal{T}$ and that each sensor in T transmits its observation back to the fusion node according to topology T. Let $\pi_{k,T}$ be the probability that transmission tree T is selected at time k. [3] Then, we may define

$$p_{k,i} \triangleq \sum_{T \in \mathcal{T}, s_i \in V_T} \pi_{k,T} \tag{12}$$

the marginal probability that sensor i is selected at time k. Moreover, we can introduced the binary random variable $\delta_{k,T}$ such that $\delta_{k,T} = 1$ if the transmission tree T is selected at time k and $\delta_{k,T} = 0$ otherwise. Similarly, define the binary random variable $\gamma_{k,i}$ to be 1 if sensor i is selected at time k and 0 otherwise. Hence, the estimation error covariance P_k and information matrix Z_k satisfy the following recursive equations:

$$P_k = P_{k|k-1} - \sum_{T \in \mathcal{T}} \delta_{k,T} P_{k|k-1} C_T' (C_T P_{k|k-1} C_T' + R_T)^{-1} C_T P_{k|k-1},$$

$$Z_k = Z_{k|k-1} + \sum_{i=1}^{m} \gamma_{k,i} \frac{C_i' C_i}{r_i}$$

where

$$P_{k|k-1} = AP_{k-1}A' + Q, Z_{k|k-1} = (AZ_{k-1}^{-1}A' + Q)^{-1}$$

Since transmission trees are randomly selected, P_k becomes a random matrix. Thus, we can only minimize the expected next step ahead estimation error covariance matrix $\mathbb{E}P_k$ and require that the expected energy consumption does not exceed the designated threshold $\mathcal{E}_d$. These considerations lead to the formulation of the following optimization problem

Problem 2 (Random Transmission Tree Selection):

$$\min_{\pi_{k,T}} \; trace(\mathbb{E}P_k)$$
$$\sum_{T \in \mathcal{T}} \pi_{k,T} \mathcal{E}(T) \leq \mathcal{E}_d,$$
$$\pi_{k,T} \geq 0, \sum_{T \in \mathcal{T}} \pi_{k,T} = 1.$$

Remark 3: The main difference between Problem 2 and Problem 1 is that instead of optimizing on a discrete space $\mathcal{T}$, we are optimizing over $\pi_{k,T}$, which is continuous. This brings two advantages. First one can see that if we add the constraint that $\pi_{k,T}$ is either 0 or 1, then Problem 2 and Problem 1 are equivalent. Hence, the optimal cost of Problem 2 will be a lower-bound to the optimal cost of Problem 1 since we now have more choices. As a result, stochastic sensor selection strategy can actually improve the performance (at least in the expected sense). The second advantage is that the feasible set $\pi_{k,T}$ is now a convex set since we remove the $0-1$ constraints.

[3] Note that $T_0 = \{\{s_0\}, \phi\}$ also belongs to $\mathcal{T}$. Hence π_{k,T_0} is the probability that no sensors are used at time k.

However there are two main drawbacks in the above formulation that still make Problem 2 intractable:

1) It is usually difficult to write $\mathbb{E}P_k$ as an explicit function of $\pi_{k,T}$.[4]
2) Since $|\mathcal{T}|$ is large, the number of optimization variables and constraints may not be polynomial with respect to the number of nodes.

In the next subsection, we will devise a possible relaxation method that allows one to overcome the above two problems.

B. Toward a Convex Optimization Formulation

As a first step to relax Problem 2 we can define an upper-bound U_k and a lower-bound L_k for $\mathbb{E}P_k$ by means of the following Theorems:

Theorem 1: Let $L_0 = P_0$ and

$$L_k = \left(L_{k|k-1}^{-1} + \sum_{i=1}^{m} p_{k,i} \frac{C_i C_i'}{r_i} \right)^{-1}, \tag{13}$$

where

$$L_{k|k-1} = A L_{k-1} A' + Q. \tag{14}$$

The following inequalities hold:

$$L_k^{-1} \geq \mathbb{E}Z_k,\ \mathbb{E}P_k \geq (\mathbb{E}Z_k)^{-1} \geq L_k. \tag{15}$$

Theorem 2: Let $U_0 = P_0$ and

$$U_k = U_{k|k-1} - \sum_{T\in\mathcal{T}} \pi_{k,T} U_{k|k-1} C_T' (C_T U_{k|k-1} C_T' + R_T)^{-1} C_T U_{k|k-1},$$

where

$$U_{k|k-1} = A U_{k-1} A' + Q.$$

The following inequality holds:

$$U_k \geq \mathbb{E}P_k.$$

The proofs of both theorems involve the use of the Jensen's inequality and is omitted due to space limit.

Compared to $\mathbb{E}P_k$, matrices U_k and L_k are deterministic and explicit functions of $\pi_{k,T}$. Moreover, note that while U_k depends on variables $\pi_{k,T}$, the number of which may be not polynomial in the number of sensors, L_k only depends on the marginal probability $p_{k,i}$. If we minimize the trace of the lower-bound L_k instead of the trace $\mathbb{E}P_k$, the problem simplifies as follows

Problem 3 (Lower-Bound for Random Tree Selection):

$$\begin{aligned} \min_{\pi_{k,T},\, p_{k,i}} \quad & trace(L_k) \\ & L_k = \left(L_{k|k-1}^{-1} + \sum_{i=1}^{m} p_{k,i} \frac{C_i' C_i}{r_i} \right)^{-1}, \\ & L_{k|k-1} = A L_{k-1} A' + Q, \\ & \sum_{T\in\mathcal{T}} \pi_{k,T} \mathcal{E}(T) \leq \mathcal{E}_d, \\ & \pi_{k,T} \geq 0,\ \sum_{T\in\mathcal{T}} \pi_{k,T} = 1, \\ & p_{k,i} \triangleq \sum_{T\in\mathcal{T}, s_i \in V_T} \pi_{k,T}. \end{aligned}$$

[4]The reader can refer to [13] for more details.

Remark 4: It is worth remarking that, once we find the marginal probability $p_{k,i}$ that minimizes the lower-bound, we can easily find a corresponding $\pi_{k,T}, \forall T \in \mathcal{T}$, and then compute the upper-bound U_k to have a measure of the tightness of the lower-bound.

The last step needed to make the problem tractable is to remove the variables $\pi_{k,T}$ from the constraints. To this end, it is easy to see that the following Propositions hold:

Proposition 1: The energy cost of a given collection of tree selection probabilities $\pi_{k,T}, \forall T \in \mathcal{T}$ is a linear function of the resulting marginal probability:

$$\sum_{T\in\mathcal{T}} \pi_{k,T} \mathcal{E}(T) = \sum_{i=1}^{m} c_i p_{k,i}. \tag{16}$$

Proposition 2: If $p_{k,i} \in [0, 1]$ with

$$p_{k,i} \leq p_{k,j}, \qquad \text{if } j \text{ is parent of } i, \tag{17}$$

then there exists at least one collection of tree selection probabilities $\pi_{k,T}, \forall T \in \mathcal{T}$[5] such that

$$\pi_{k,T} \geq 0,\ \sum_{T\in\mathcal{T}} \pi_{k,T} = 1,\ p_{k,i} \triangleq \sum_{T\in\mathcal{T}, s_i\in V_T} \pi_{k,T}.$$

The above Propositions allow us to rewrite, without loss of generality, Problem 3 as follows

Problem 4 (Lower-Bound for Random Tree Selection):

$$\begin{aligned} \min_{p_{k,i}} \quad & trace(L_k) \\ & L_k = \left(L_{k|k-1}^{-1} + \sum_{i=1}^{m} p_{k,i} \frac{C_i' C_i}{r_i} \right)^{-1}, \\ & L_{k|k-1} = A L_{k-1} A' + Q, \\ & 0 \leq p_{k,i} \leq 1,\ i = 1, \ldots, m, \\ & p_{k,i} \leq p_{k,j}, \qquad \text{if } j \text{ is parent of } i. \end{aligned}$$

The above problem is a convex optimization problem with a number of variables and constraints which is a polynomial function of the dimension of the original problem. Thus, it is solvable in a polynomial time.

C. Implementation

In this section we discuss a possible implementation of our sensor selection algorithm. We will assume that, since the optimization does not depend on the real-time sensor measurement y_k, the optimization step is performed off-line in a centralized fashion. Then, each sensor i will store the optimal $p_{1,i}, p_{2,i}, \ldots, p_{k,i}$ for a long enough time horizon[6]. Moreover, we assume that sensor i will also store $p_{1,j}, \ldots, p_{k,j}$ for all its children. As a result, we do not need to communicate $p_{k,i}$ in real-time.

At each time k, we have to select one subset of sensors according to marginal probabilities $p_{k,1}, \ldots, p_{k,m}$. However we do not want the fusion center to query the nodes since

[5]It is worth noticing that in general there may exist more than one set of $\pi_{k,T}, \forall T \in \mathcal{T}$ with the same marginal probabilities.

[6]Storage is usually cheap for sensor networks. Moreover, it is also possible to compress the sequence $p_{1,i}, \ldots, p_{k,i}$ to save storage, especially when $p_{k,i}$ converges as $k \to \infty$.

this would increase communication overhead, thus defying the purpose of sensor selection. To overcome this problem, we propose the following algorithm:

1) Every sensor is equipped with the same random number generator and same seed at time 0.
2) At time k, each sensor draws a random number α_k from the random number generator.
3) If sensor i has no children, then it will compare α_k with $p_{k,i}$. If $\alpha_k \leq p_{k,i}$, then it will transmit the measurement to its parent. Otherwise it will not transmit.
4) If sensor i has children, then it will compare α_k with $p_{k,j}$, where j is the index of its child node. If $\alpha_k \leq p_{k,j}$, then sensor i knows that child j will forward an observation packet to him. After i receives all the observation packets from its children, it will merge all the packets and its own observations into one single packet and forward it to its parent. If $\alpha_k > p_{k,j}$ for all j child of i, then i will compare α_k with $p_{k,i}$. If $\alpha_k \leq p_{k,i}$, then sensor i transmits its measurements to its parent. Otherwise it will not transmit.

Since every sensor is equipped with the same random number generator and same seed, they will get the same α_k at time k. Hence, the above algorithm will guarantee that all sensors agree on same transmission topology T which satisfies the marginal distribution $p_{k,i}$. It is worth to remark that in such a scheme the only communication needed is the transmission of observation packets and no communication for coordination purpose is needed.

IV. Simulation Result

For the sake of showing the effectiveness of the proposed method we apply our stochastic sensor selection algorithm to a numerical example in which a sensor network is deployed to monitor a diffusion process in a planar closed region. The model of the diffusion process in a planar closed region can be modelled as

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} \right), \tag{18}$$

with boundary conditions

$$\left.\frac{\partial u}{\partial x_1}\right|_{t,0,x_2} = \left.\frac{\partial u}{\partial x_1}\right|_{t,l,x_2} = \left.\frac{\partial u}{\partial x_2}\right|_{t,x_1,0} = \left.\frac{\partial u}{\partial x_2}\right|_{t,x_1,l} = 0, \tag{19}$$

where x_1, x_2 indicate the coordinates of the region; $u(t, x_1, x_2)$ denotes the temperature at time t at location (x_1, x_2) and α indicates the speed of the diffusion process. We use the finite difference method to discretize this model and we assume that the planar region is a square of side l meters. As a result, a $N \times N$ grid is achieved with grid step $h = l/(N-1)$. We also sample the system in time with frequency of 1 Hz. Applying finite difference method, equation (18) becomes

$$\begin{aligned} u(k+1,i,j) - u(k,i,j) &= \alpha/h^2[u(k,i-1,j) + u(k,i,j-1) \\ &+ u(k,i+1,j) + u(k,i,j+1) - 4u(k,i,j)], \end{aligned} \tag{20}$$

where $u(k, i, j)$ denotes the temperature at time k, at location (ih, jh).

Remark 5: If either i or j is greater than N or less than 0, then the location is outside the grid. Then, $u(k, i, j)$ is replaced by the value of its nearest neighbour in the grid as a consequence of the boundary conditions we imposed.

If we group all the temperature values at k in the vector $U_k = [u(k,0,0), \ldots, u(k,0,N-1), u(k,1,0), \ldots, u(k,N-1,N-1)]^T$, we can write the evolution of the discretized system as $U_{k+1} = AU_k$, where the A matrix can be calculated from (20). If we introduce process noise, U_k will evolve according to

$$U_{k+1} = AU_k + w_k, \tag{21}$$

where $w_k \in \mathcal{N}(0,\ Q)$. We assume that the fusion center is located in the center of the room $(2, 2)$. We also assume that m sensors are randomly distributed in the region and each sensor measures a linear combination of temperature of the grid around it. In particular, indicate the location of sensor $\hat{s}$ with $(\hat{x}_1, \hat{x}_2)$ such that $\hat{x}_1 \in [i, i+1)$ and $\hat{x}_2 \in [j, j+1)$. Define $\Delta\hat{x}_1 = \hat{x}_1 - i$ and $\Delta\hat{x}_2 = \hat{x}_2 - j$. We also assume that the measurement of the sensor is

$$\begin{aligned} y_{\hat{s}} = [\,&(1-\Delta\hat{x}_1)(1-\Delta\hat{x}_2)u(k,i,j) + \\ &\Delta\hat{x}_1(1-\Delta\hat{x}_2)u(k,i+1,j) + \\ &(1-\Delta\hat{x}_1)\Delta\hat{x}_2 u(k,i,j+1) + \\ &\Delta\hat{x}_1\Delta\hat{x}_2 u(k,i+1,j+1)\ ]/h^2 + noise. \end{aligned} \tag{22}$$

Indicating with Y_k the vector of all the measurements at time k, it follows that:

$$Y_k = CU_k + v_k, \tag{23}$$

where v_k denotes the measurement noise at time k assumed to have normal distribution $\mathcal{N}(0,\ R)$ and C is the observation matrix that can be computed from (22). Finally, we assume that the sensor network is fully connected and the communication cost from sensor i to j is

$$cost(e_{i,j}) = c + d_{i,j}^2$$

where d_{ij} is the Euclidean distance from sensor i to sensor j and c is a constant related to the sensing energy consumption[7]. We then compute the minimum spanning tree of the graph and force sensor to only communicate with its parent node in the minimum spanning tree. For the simulations, we impose the following values for the parameters:

- $m = 20$, $\alpha = 0.1\ m^2/s$, $l = 4\ m$ and $N = 5$.
- $Q = I$, $R = I$, $\Sigma = 4I$.
- $\mathcal{E}_d = 5$,$c = 1$.

Figure 1 shows the evolution trace of upper-bound, lower-bound and expected error covariance. The expected error covariance is computed by averaging 100 sample paths. It is easy to see that the upper-bound is about 15% higher than the lower-bound, which means that the lower-bound is quite tight. Figure 2 shows the trace of P_k for the optimal solution of Problem 1, which is found by exhaustive search, together

[7] c models the fact that as the distance goes to zero the communication cost is non negligible even when sensors are infinitely close

with the trace of P_k from a sample path of the stochastic schedule and the EP_k of the stochastic schedule. The optimal deterministic schedule is worse than EP_k, which highlights the fact that the stochastic formulation, as discussed in Remark 3, is better than the deterministic formulation in the expected sense. It is also worth noticing that the computation time of Problem 4 is 1.8 seconds on Intel Core2 2GHz CPU using Matlab R2006b and CVX 1.2, while the exhaustive search takes 104 seconds to be completed.

V. Conclusions

In this paper, we propose a stochastic sensor selection algorithm for a tree topology wireless sensor network. We solve the optimal stochastic sensor selection problem after convex relaxation of the optimization problem. We also provide a possible implementation of our random sensor selection algorithm without introducing any communication overhead. Future works will aim at finding the optimal stochastic sensor schedule for arbitrary topologies and designing the optimal scheduling policy that minimizes the asymptotic expected estimation error covariance matrix.

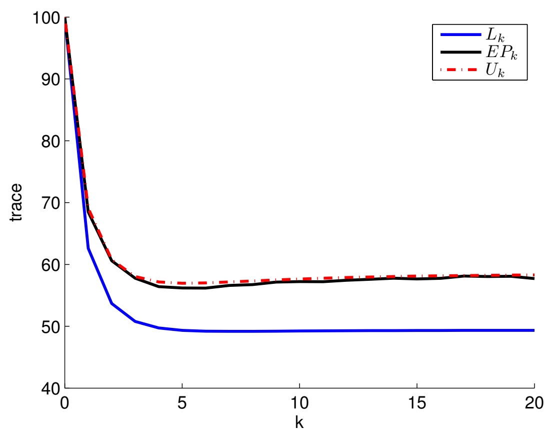

Fig. 1. Evolution of the Trace of U_k, L_k and EP_k

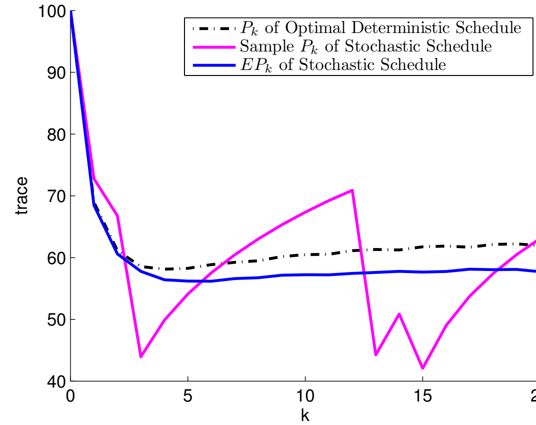

Fig. 2. Evolution of the Trace of P_k Given by Optimal Deterministic Schedule for Problem 1, Sample P_k for Stochastic Schedule, EP_k for Stochastic Schedule

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