

# Distributed Coordination Strategies for Interconnected Multi-Agent Systems

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**Abstract:** In this paper we present a novel distributed supervision strategy for multi-agent linear systems connected via data networks and subject to coordination constraints. Such a *coordination-by-constraint* paradigm is based on an on-line active set-point management and is characterized by a set of spatially distributed dynamic systems, connected via communication channels, with possibly dynamical coupling amongst them which need to be supervised and coordinated in order to accomplish their overall objective. Two distributed strategies will be fully described and analyzed. First, we will propose a "sequential" distributed strategy in which only one agent at the time is allowed to manipulate its own reference signal. Such a strategy will be instrumental to introduce a more effective "parallel" distributed strategy, in which all agents are allowed, under certain conditions, to modify their own reference signals simultaneously. To show the effectiveness of the proposed methods, the distributed coordination of dynamically coupled autonomous vehicles under input-saturation and formation accuracy constraints is presented.

Keywords: Nonlinear Control, Distributed Predictive Control, Command Governor.

## 1. INTRODUCTION

The problem of interest here is the design of distributed supervision strategies based on Command Governor (CG) ideas for multi-agent systems where the use of a centralized coordination unit is impracticable because requiring unrealistic or unavailable communication infrastructures. From a theoretical point of view, distributed control policies for dynamically coupled systems have been recently studied in (Camponogara et al. [2002], D'Andrea and Dullerud [2003], Venkat et al. [2005], Magni and Scattolini [2006], Dunbar [2007]). The CG approach (see Gilbert et al. [1995], Casavola et al. [2000, 2006]) is a well known and established methodology that provides a simple and effective way to enforce pointwise-in-time constraints along the trajectories of a closed-loop system. The CG is a nonlinear device which is added to a pre-compensated control system. Whenever necessary, the CG modifies the reference to the closed-loop system so as to avoid constraint violations. In this paper, we will make use of a recently proposed solution (see Garone *et al.* [2009], Garone et al. [2010], Casavola et al. [2009]) to the CG problem, hereafter referred to as the *Feed Forward* CG (FF-CG) approach, that, at the price of some additional conservativeness, is able to accomplish the CG task in the absence of an explicit measure of the state. The idea behind such an approach is that, if sufficiently smooth transitions in the set-point modifications are acted by the CG unit, one can have a high confidence on the expected value of the state, even in the absence of an explicit measure of it, thanks to the asymptotical stability of the system. Clearly, the scheme acts in a feed-forward way but the reference modification is implicitly undertaken on the basis of the *expected* value of the current state *inferred* by the knowledge of the currently applied constant set-point and the constraints

are always satisfied. A detailed investigation on the property and performance differences between feedback and feedforward centralized CG supervising strategies in the presence of disturbances is presented in (Garone et al. [2010]). The peculiarities of the FF-CG scheme make it an attractive solution for distributed frameworks because they alleviate the need to make the entire aggregate state, or substantially parts of it, known to all agents at each time instant, the latter being unrealistic or requiring unrealistic communication infrastructures in some large scale applications. This paper introduces two novel distributed schemes, which are direct improvements of those presented in (Garone *et al.* [2009]) and are expected to require lower communication rates than other distributed approaches for their implementation, e.g those based on consensus mechanisms. The feasibility and stability properties of the presented approaches will be also discussed.

## 2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

The aim of this paper is to develop a distributed CG strategy for interconnected linear systems subject to convex set-membership input and state-related constraints. To this end, consider a set of  $N$  subsystems  $\mathcal{A} = \{1, \dots, N\}$ . Each subsystem is a LTI closed-loop dynamical system regulated by a local controller which ensures stability and good closed-loop properties when the constraints are not active (small-signal regimes when the coordination is effective). Let the  $i$ -th closed-loop subsystem be described by the following discrete-time model

$$\begin{cases} x_i(t+1) = \Phi_{ii}x_i(t) + G_i^g g_i(t) + \sum_{j \in \mathcal{A} - \{i\}} \Phi_{ij}x_j(t) \\ y_i(t) = H_i^y x_i(t) \\ c_i(t) = H_i^c x_i(t) + L_i g(t) \end{cases} \quad (1)$$

where:  $t \in \mathbb{Z}_+$ ,  $x_i \in \mathbb{R}^{n_i}$  is the state vector (which includes the controller states under dynamic regulation),  $g_i \in \mathbb{R}^{m_i}$

the manipulable reference vector which, if no constraints (and no CG) were present, would coincide with the desired reference  $r_i \in \mathbb{R}^m$  and  $y_i \in \mathbb{R}^{m_i}$  is the output vector which is required to track  $r_i$ . Finally,  $c_i \in \mathbb{R}^{n_i^c}$  represents the local constrained vector which has to fulfill the set-membership constraint

$$c_i(t) \in \mathcal{C}_i, \forall t \in \mathbb{Z}_+, \quad (2)$$

$\mathcal{C}_i$  being a convex and compact polytopic set. It is worth pointing out that, in order to possibly characterize global (coupling) constraints amongst states of different subsystems, the vector  $c_i$  in (1) is allowed to depend on the aggregate state and manipulable reference vectors  $x = [x_1^T, \dots, x_N^T]^T \in \mathbb{R}^n$ , with  $n = \sum_{i=1}^N n_i$ , and  $g = [g_1^T, \dots, g_N^T]^T \in \mathbb{R}^m$ , with  $m = \sum_{i=1}^N m_i$ . Moreover, we denote by  $r = [r_1^T, \dots, r_N^T]^T \in \mathbb{R}^m$ ,  $y = [y_1^T, \dots, y_N^T]^T \in \mathbb{R}^m$  and  $c = [c_1^T, \dots, c_N^T]^T \in \mathbb{R}^{n^c}$ , with  $n^c = \sum_{i=1}^N n_i^c$ , the other relevant aggregate vectors. The overall system arising by the composition of the above  $N$  subsystems can be described as

$$\begin{cases} x(t+1) = \Phi x(t) + Gg(t) \\ y(t) = H^y x(t) \\ c(t) = H^c x(t) + Lg(t) \end{cases} \quad (3)$$

where

$$\Phi = \begin{pmatrix} \Phi_{11} & \dots & \Phi_{1N} \\ \vdots & \ddots & \vdots \\ \Phi_{N1} & \dots & \Phi_{NN} \end{pmatrix}, G = \begin{pmatrix} G_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & G_N \end{pmatrix}$$

$$H^y = \begin{pmatrix} H_1^y & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & H_N^y \end{pmatrix}, H^c = \begin{pmatrix} H_1^c \\ \dots \\ H_N^c \end{pmatrix}, L = \begin{pmatrix} L_1 \\ \dots \\ L_N \end{pmatrix}.$$

It is further assumed that

**A1.** The overall system (3) is asymptotically stable.

**A2.** System (3) is off-set free i.e.  $H^y(I_n - \Phi)^{-1}G = I_m$ .

Roughly speaking, the CG design problem we want to solve is that of locally determine, at each time step  $t$  and for each agent  $i \in \mathcal{A}$  associated to each subsystem, a suitable reference signal  $g_i(t)$  which is the best approximation of  $r_i(t)$  such that its application never produces constraints violation, i.e.  $c_i(t) \in \mathcal{C}_i, \forall t \in \mathbb{Z}_+, i \in \mathcal{A}$ .

Classical centralized solutions of the above stated CG design problem (see Casavola et al. [2000, 2006]) have been achieved by finding, at each time  $t$ , a CG action  $g(t)$  as a function of the current reference  $r(t)$  and measured state  $x(t)$

$$g(t) := \underline{g}(r(t), x(t)) \quad (4)$$

such that  $g(t)$  is the best approximation of  $r(t)$  under the condition  $c(t) \in \mathcal{C}$ , where  $\mathcal{C} \subseteq \{\mathcal{C}_1 \times \dots \times \mathcal{C}_N\}$  is the global admissible region. Here we will focus on a slight different approach in which the explicit dependence on the state vector disappears. This is a convenient solution to be used in a distributed environment because it eliminates the need to share the state vector amongst the agents that, as well known (see Dunbar [2007]), is one of the main difficulties in implementing distributed schemes. Such an approach, hereafter referred to as *Feed Forward CG* (FF-CG), will be described in the next Section.

### 3. THE FEED FORWARD CG APPROACH

In this section we will introduce the FF-CG as a possible solution to the CG problem. Let us introduce, for a given  $\delta > 0$ , the sets:

$$\begin{aligned} \mathcal{C}^\delta &:= \mathcal{C} \sim \mathcal{B}_\delta \\ \mathcal{W}^\delta &:= \{g \in \mathbb{R}^m : c_g \in \mathcal{C}^\delta\} \end{aligned} \quad (5)$$

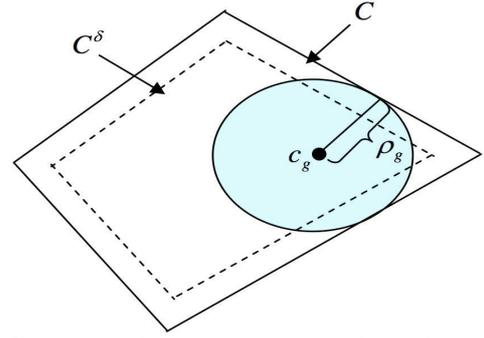


Fig. 1. Geometrical representation of condition (8) for  $c \in \mathbb{R}^2$

where  $\mathcal{B}_\delta$  is the ball of radius  $\delta$  centered at the origin and  $\mathcal{A} \sim \mathcal{E}$  is the Pontryagin set difference defined as  $\{a : a + e \in \mathcal{A}, \forall e \in \mathcal{E}\}$ . In particular,  $\mathcal{W}^\delta$ , which we assume non-empty, is the convex and closed set of all constant commands  $g$  whose corresponding equilibrium points  $c_g := H^c(I_n - \Phi)^{-1}Gg + Lg$  satisfy the constraints with margin  $\delta$ . Let introduce also the *virtual evolutions* of the  $c$ -variable

$$\hat{c}(k, x(t), g(t)) := H^c \left( \Phi^k x(t) + \sum_{i=0}^{k-1} \Phi^{k-i-1} Gg(t) \right) + Lg(t) \quad (6)$$

along the *virtual time*  $k$ , from the initial condition  $x(t)$  at time  $k = 0$  under the application of a constant command  $g(t), \forall k$ . The virtual  $c$ -variable evolution (6) can be rewritten as the sum of two amounts: a steady-state component represented by  $c_{g(t)}$  and the transient evolution  $H^c \Phi^k(x(t) - x_{g(t)})$ :

$$\hat{c}(k, x(t), g(t)) = c_{g(t)} + H^c \Phi^k(x(t) - x_{g(t)}). \quad (7)$$

Because  $g(t) \in \mathcal{W}^\delta$  and, in turn,  $c_{g(t)} \in \mathcal{C}^\delta$  at each time  $t$ , then, as also depicted in Figure 1, a sufficient condition to ensure that the constraints are satisfied, although in a quite arbitrary and conservative way, is to ensure that the transient component is confined into a ball of radius  $\rho_{g(t)}$

$$\|H^c \Phi^k(\hat{x}(t) - x_{g(t)})\| \leq \rho_{g(t)}, \forall k \geq 0 \quad (8)$$

where  $\rho_{g(t)}$  represents the minimum distance between  $c_{g(t)}$  and the border of  $\mathcal{C}$

$$\begin{aligned} \rho_g &:= \arg \max_{\rho} \rho \\ &\text{subject to } \mathcal{B}^\rho(c_g) \subseteq \mathcal{C}. \end{aligned} \quad (9)$$

where  $\mathcal{B}^\rho(c_g)$  represents the ball of radius  $\rho$  centered at  $c_g$ . Assuming  $\mathcal{C}$  a polytope,  $\rho_g$  can be easily computed as the minimal distance between a point and a set of hyperplanes and, then, selected as the minimum of a collection of  $g$ -valued linear functions. Details can be found in (Garone et al. [2010]) and (Casavola et al. [2009]). Please note that, by construction,  $\rho_g \geq \delta, \forall g \in \mathcal{W}^\delta$ .

Then, the CG design problem translates into the problem of defining an algorithm that is able to select, at each time  $t$ , a reference value  $g(t)$  such that (8) holds true for all  $k \geq 0$ . In the absence of state availability, a possible approach to deal with this problem is based on the idea that, a sufficient long time after the application of a new FF-CG command, the transient contribution decreases and can be bounded within a certain percentage of its initial magnitude. More formally, let us introduce the following notion of *Generalized Settling time*:

**Definition (Generalized Settling Time)** - The integer  $\tau > 0$  is said to be a *Generalized Settling Time with parameter*  $\gamma$ , with  $0 < \gamma < 1$ , for the pair  $(H^c, \Phi)$ , if

$$\|H^c\Phi^k x\| \leq M(x), \quad k = 0, \dots, \tau - 1 \quad (10)$$

$$\|H^c\Phi^{\tau+k} x\| \leq \gamma M(x), \quad \forall k \geq 0$$

holds true for each  $x \in \mathbb{R}^n$  with the real  $M(x) > 0$  any arbitrarily chosen upper-bound.  $\square$

Such a definition (see Garone et al. [2010] for details) implies that if at time  $t - \tau$  a certain command  $g(t - \tau)$  was applied a sufficient condition to ensure constraints satisfaction is to select a new command  $g(t)$  such that the following inequality is satisfied

$$\|H^c\Phi^k(\hat{x}(t) - x_{g(t)})\| \leq \rho_{g(t)}, \quad \forall k \geq 0. \quad (11)$$

By introducing the  $\tau$ -step incremental reference  $\Delta g(t) = g(t) - g(t - \tau)$ , and by observing that  $x_{\Delta g(t)} = x_{g(t)} - x_{g(t-\tau)}$ , a sufficient condition to ensure (11) is

$$\|H^c\Phi^k x_{\Delta g(t)}\| \leq \rho_{g(t-\tau) + \Delta g(t)} - \gamma \rho_{g(t-\tau)}, \quad \forall k \geq 0. \quad (12)$$

Finally we can formulate the Feed Forward CG selection algorithm as follows

#### The FF-CG Algorithm

REPEAT AT EACH TIME  $t$

1.1 IF ( $t == \kappa\tau, \kappa = 1, 2, \dots$ )

$$1.1.1 \text{ SOLVE } g(t) = \arg \min_g \|g - r(t)\|_{\Psi}^2 \quad (13)$$

$$\text{SUBJECT TO : } \begin{cases} g \in \mathcal{W}^\delta \\ (g - g(t - \tau)) \in \Delta \mathcal{G}_{g(t-\tau)} \end{cases} \quad (14)$$

1.2 ELSE  $g(t) = g(t - 1)$

2.1 APPLY  $g(t)$

where  $\Phi > 0$  is a weighting matrix and  $\Delta \mathcal{G}_g$  is defined as the set of all the possible  $\tau$ -step incremental commands from  $g$  ensuring inequality (12) to hold true

$$\Delta \mathcal{G}_g = \{\Delta g : \|H^c\Phi^k(I - \Phi)^{-1}G\Delta g\| \leq \rho_{g+\Delta g} - \gamma\rho_g, \forall k \geq 0\}. \quad (15)$$

It is worth to note that the sets  $\mathcal{W}^\delta$  and the generalized settling time  $\tau$  can be computed off-line from the outset. Finally the following properties can be proved

**Proposition 1.** - Let assumptions **A1-A2** be fulfilled. Consider system (3) along with the **FF-CG** selection rule and let an admissible command signal  $g(0) \in \mathcal{W}^\delta$  be applied at  $t = 0$  such that (8) holds true. Then:

- (1) the minimizer in (13) uniquely exists (computed every  $\tau$  steps) and can be obtained by solving a convex constrained optimization problem;
- (2) the constraints are fulfilled for all  $t \in \mathbb{Z}_+$ ;
- (3) the overall system is asymptotically stable and whenever  $r(t) \equiv r$ , the sequence of  $g(t)$  converges in finite time either to  $r$  or to its best steady-state admissible approximation:  $g(t) \rightarrow \hat{r} := \arg \min_{g \in \mathcal{W}^\delta} \|g - r\|_{\Psi}^2$ .  $\square$

For details please refer to (Casavola et al. [2009]).

#### 4. DISTRIBUTED FF-CG

Here we introduce two distributed FF-CG schemes based on the presented FF-CG approach assuming that the agents are connected via a communication network. Such a network is modeled as a *communication graph*: an undirected graph  $\mathcal{G} = (\mathcal{A}, \mathcal{B})$ , where  $\mathcal{A}$  denotes the set of the  $N$  subsystems and  $\mathcal{B} \subset \mathcal{A} \times \mathcal{A}$  the set of edges representing the communication links amongst agents. More precisely, the edge  $(i, j)$  belong to  $\mathcal{B}$  if and only if the agents governing the  $i$ -th and the  $j$ -th subsystems are able to directly share information within  $\tau$  sampling times. The communication graph is assumed to be connected, i.e. for each couple of agents  $i \in \mathcal{A}, j \in \mathcal{A}$  there exists at least one

sequence of edges connecting  $i$  and  $j$ , with the minimum number of edges connecting the two agents denoted by  $d_{i,j}$ . The set of all agents with a direct connection with the  $i$ -th agent will be referred to as *Neighborhood of the  $i$ -th agent*  $\mathcal{N}_i = \{j \in \mathcal{A} : d_{i,j} = 1\}$ .

##### 4.1 Sequential Procedure (S-FFCG)

Let  $\mathcal{G}$  be a Hamiltonian graph and assume, without loss of generality, that the sequence  $\mathcal{H} = \{1, 2, \dots, N - 1, N\}$  is a Hamiltonian cycle. The idea behind the approach is that only one agent at decision time is allowed to manipulate its local command signal  $g_i(t)$  while all others are instructed to hold their previous values. After each decision, the agent in charge will update the global command received from the previous updating agent and will forward this new value to the next updating agent in the cycle. Such a polling policy implies that, eventually after a preliminary initialization cycle, at each time instant the "agent in charge" always knows the whole aggregate vector  $g(t - \tau)$ . By exploiting this observation we can define the following distributed FF-CG algorithm:

##### Sequential-FFCG Algorithm (S-FFCG) - Agent $i$

REPEAT AT EACH TIME  $t$

1.1 IF ( $t == \kappa\tau, \kappa = 0, 1, \dots$ ) && ( $\kappa \bmod N == i$ )

1.1.1 RECEIVE  $g(t - \tau)$  FROM THE PREVIOUS AGENT IN THE CYCLE  $\mathcal{H}$

$$1.1.2 \text{ SOLVE } g_i(t) = \arg \min_{g_i} \|g_i - r_i(t)\|_{\Psi_i}^2$$

SUBJECT TO :

$$\begin{cases} g(t) = [g_1^T(t - \tau), \dots, g_i^T, \dots, g_N^T(t - \tau)]^T \in \mathcal{W}^\delta \\ (g_i - g_i(t - \tau)) \in \Delta \mathcal{G}_{g_i}^0 \end{cases} \quad (16)$$

1.1.3 APPLY  $g_i(t)$

1.1.4 UPDATE  $g(t) = [g_1^T(t - \tau), \dots, g_i(t), \dots, g_N^T(t - \tau)]^T$

1.1.5 TRANSMIT  $g(t)$  TO THE NEXT AGENT IN  $\mathcal{H}$

1.2 ELSE

1.2.1 APPLY  $g_i(t) = g_i(t - 1)$

where  $\Psi_i > 0$  is a weighting matrix,  $\kappa \bmod N$  is the remainder of the integer division  $\kappa/N$  and

$$\Delta \mathcal{G}_{g_i}^0 = \{\Delta g_i \mid [0, 0, \dots, \Delta g_i^T, \dots, 0]^T \in \Delta \mathcal{G}_g\}$$

is the set of all possible command variations for  $g_i$  in the case that the commands of all other agents are frozen.

**Proposition 2.** - Let assumptions **A1-A2** be fulfilled. Consider system (3) as the composition of  $N$  subsystems in form (1) along with the distributed **S-FFCG** selection rule and let an admissible aggregate command signal  $g(0) = [g_1^T(0), \dots, g_N^T(0)]^T \in \mathcal{W}^\delta$  be applied at  $t = 0$  such that (8) holds true. Then

- (1) for each agent  $i \in \mathcal{A}$  the algorithm produces at each time step a feasible local command  $g_i(t)$ ;
- (2) the constraints are fulfilled for all  $t \in \mathbb{Z}_+$ ;
- (3) the overall system is asymptotically stable. In particular, whenever  $r(t) \equiv r$ , the sequence of the aggregate vectors  $g(t) = [g_1^T(t), \dots, g_N^T(t)]^T$  converges in finite time either to  $r$  or to an admissible approximation in  $\mathcal{W}^\delta$  which is Pareto-optimal w.r.t. the local objective functionals  $\|g_i - r_i\|_{\Psi_i}^2, i = 1, \dots, N$ .  $\square$

**Remark 1** - A proof of Proposition (2) is here omitted for brevity. See (Casavola et al. [2009]) for details.  $\square$

##### 4.2 Parallel FF-CG (P-FFCG)

The main drawback of the S-FFCG algorithm is that, every  $\tau$  time instants, only one agent at a time is allowed to modify its local command. This yields to a reduced

capability to track the desired reference  $r(t)$ . In order to overcome such a limitation, any agent should be enabled to select its local command every  $\tau$  time instants. The two key points to be considered in building up such a kind of strategy are:

- a) the definition of the information set available to each agent
- b) the determination of a set of decentralized "selection rules" such that the composition of all feasible local commands satisfies global constraints (14).

With regard to the information available to each agent, we will assume that each agent acts as a gateway in redistributing data amongst the other no directly connected agents. Then, at each time instant  $t$ , the  $i$ -th agent has knowledge of the following vector:

$$\xi_i(t-\tau) = [g_1^T(t-d_{i,1}\tau), \dots, g_i^T(t-\tau), \dots, g_N^T(t-d_{i,N}\tau)]^T.$$

It results that the most recent information on the applied commands shared by all agents at each decision time  $t$  is given by the vector

$$\xi(t-\tau) = [g_1^T(t-d_{max,1}\tau), \dots, g_N^T(t-d_{max,N}\tau)]^T$$

where  $d_{max,i} = \max_{j \in \mathcal{A}d_{i,j}}$ .

The main idea behind the proposed selection rule is that of generating, every  $\tau$  steps and on the basis of the information shared by all the agents in the network, a set of decoupled alternative constraints (one for each agent) such that their local fulfilment implies the fulfilment of global constraints (14). In other words, at each computation time step, we will substitute the admissible region for  $\Delta g$  given by conditions (14) with its *set Cartesian decomposition* (Bertsekas [2007]).

If such a decomposition is opportunely performed, the problem decouples and each agent will have simply to fulfil the inclusion into a local set in the form

$$\Delta g_i(t) \in \Delta G_{v,i}(t), i = 1, \dots, N \quad (17)$$

with  $\Delta G_{v,i}(t) \subseteq \mathbb{R}^{n_i^c}$ ,  $i = 1, \dots, N$  convex and compact sets containing  $0_{n_i^c}$  for all  $t \geq 0$ .

It remains to understand how to generate local decoupled constraints that ensure global constraints satisfaction. The first step is to observe that if constraints (17) are satisfied at each time step, then we can define the feasible set of all possible actual values of the command  $g(t)$  computed on the basis of the common information vector  $\xi(t-\tau)$  as follows

$$\begin{aligned} \Xi(t-\tau) = & \{\xi(t-\tau)\} + \\ & + \left( (\Delta G_{v,1}(t-\tau) \times \dots + \Delta G_{v,1}(t-d_{max,1}\tau)) \times \dots \right. \\ & \left. \dots \times (\Delta G_{v,N}(t-\tau) + \dots + \Delta G_{v,N}(t-d_{max,1}\tau)) \right) \end{aligned} \quad (18)$$

where  $+$  denotes the Pontryagin set sum defined as  $X + Y = \{z = x + y : \forall x \in X, \forall y \in Y\}$ . On the basis of the above feasible values of  $g(t-\tau)$ , the set of the admissible aggregate command variations can be easily computed as follows

$$\begin{aligned} \Delta \mathcal{G}_{\Xi(t-\tau)}^{\mathcal{W}_\delta} = & \{\Delta g | \Delta g \in \Delta \mathcal{G}_g, \forall g \in \Xi(t-\tau)\} \\ & \cap \{\Delta g | (g + \Delta g) \in \mathcal{W}_\delta, \forall g \in \Xi(t-\tau)\} \end{aligned} \quad (19)$$

Finally, the (approximated) Cartesian decomposition generating the agent-wise decoupled constraints (17) should satisfy the following set inclusion condition

$$\Delta G_{v,1}(t) \times \dots \times \Delta G_{v,N}(t) \subseteq \Delta \mathcal{G}_{\Xi(t-\tau)}^{\mathcal{W}_\delta}, \quad (20)$$

Note also that, because  $\Xi(t-\tau)$  is a common information shared by all agents in the network, then, once an opportune objective function is defined, all agents will be able to independently determine the same collection of sets

$\Delta G_{v,i}(t)$ ,  $i = 1, \dots, N$ . We can finally describe the Parallel FF-CG procedure to be performed every  $\tau$  steps:

- 1.1 Each agent determines the collection of sets  $\Delta G_{v,i}(t)$ ,  $i = 1, \dots, N$  as the solution of own instance the following optimization problem

$$\max_{\Delta G_{v,i}(t), i=1, \dots, N} V(\Delta G_{v,1}(t) \times \dots \times \Delta G_{v,N}(t)) \quad (21)$$

subject to (20)

where  $V(\cdot)$  denotes a possible measure of the volume of a set (to achieve good dynamical properties we want  $(\Delta G_{v,1}(t) \times \dots \times \Delta G_{v,N}(t))$  to be as large as possible)

- 2.1 if  $V(\Delta G_{v,1}(t) \times \dots \times \Delta G_{v,N}(t))$  is bigger then a pre-determined threshold  $\epsilon_{thr}$ , each agent can choose its own reference by solving the following convex optimization problem

$$g_i(t) = \arg \min \|g_i - r_i(t)\|_{\Psi_i}^2 \quad (22)$$

subject to  $(g_i - g_i(t-\tau)) \in \Delta G_{v,i}(t)$

- 2.2 otherwise, after an opportune initialization phase, all agents simultaneously change the strategy and *go sequentially* under the S-FFCG strategy, where the agent in charge is constrained inside a pre-determined virtual constraint  $\Delta G_{seq,i}$ ,  $i = 1, \dots, N$ .

The threshold  $\epsilon_{thr}$  and the pre-determined sets  $\Delta G_{seq,i}$ ,  $i = 1, \dots, N$  are design parameters and have to be chosen accurately in order to ensure good dynamical performance during the switching phases.

More formally we present the following "parallel" version of the distributed FF-CG algorithm:

**Parallel-FFCG Algorithm (P-FFCG) - Agent  $i$**

REPEAT AT EACH TIME  $t$

- 1.1 IF  $(t == \kappa\tau, \kappa = 0, 1, \dots)$

- 1.1.1 SOLVE (21)

- 1.1.2 IF  $V(\Delta G_{v,1}(t) \times \dots \times \Delta G_{v,N}(t)) \geq \epsilon_{thr}$

SOLVE (22)

$\mu = 1$ ;

- ELSE IF  $\kappa \bmod N \neq i$

$$\Delta G_{v,i}(t) = \{0\}, \quad i \neq \kappa \bmod N$$

$$\Delta G_{v,i}(t) = \Delta G_{seq,i}, \quad i = \kappa \bmod N$$

$$g_i(t) = g_i(t-\tau)$$

$\mu ++$ ;

- ELSE

$$g_i(t) = \arg \min \|g_i - r_i(t)\|_{\Psi_i}^2$$

SUBJECT TO :

$$\begin{cases} (g_i - g_i(t-\tau)) \in \Delta \mathcal{G}_i^0 \\ (g_i - g_i(t-\tau)) \in \Delta G_{seq,i} \end{cases} \quad (23)$$

$$g(t) = [\bar{g}_1^T, \dots, g_i^T, \dots, \bar{g}_N^T(t-\tau)]^T \in \mathcal{W}^\delta$$

$$\forall \bar{g}_j \in \bar{\mathcal{G}}_{i,j}(\xi_i(t-\tau), \mu)$$

$$\Delta G_{v,i}(t) = \{0\}, \quad i \neq \kappa \bmod N$$

$$\Delta G_{v,i}(t) = \Delta G_{seq,i}, \quad i = \kappa \bmod N$$

$\mu ++$ ;

- 1.1.2 APPLY  $g_i(t)$

- 1.1.3 UPDATE  $\xi_i(t)$

- 1.1.4 TRANSMIT  $\xi_i(t)$  TO NEIGHBORHOOD  $\mathcal{N}_i$

- 1.1.5 RECEIVE  $\xi_j(t)$  FROM ALL NEIGHBORS  $j \in \mathcal{N}_i$

- 1.2 ELSE

- 1.2.1 APPLY  $g_i(t) = g_i(t-1)$

where  $\mu$  is a counter initialized as  $\mu = 1$ , which is incremented for each time passed in the sequential mode, and  $\bar{\mathcal{G}}_{i,j}(\xi_i(t-\tau), \mu)$  is the set of all possible values that the  $j$ -th command could have assumed from the  $i$ -th agent viewpoint:

$$\begin{aligned} \bar{\mathcal{G}}_{i,j}(\xi_i(t-\tau), \mu) \\ = \{\bar{g}_j | \bar{g}_j - g_j(t-d_{i,j}\tau) \in (\min\{0, d_{i,j} - \mu\}) \Delta \mathcal{G}_j\} \end{aligned} \quad (24)$$

Please note that the introduction of such a set and of associate conditions in the optimization problem (23) forces the initialization step of the sequential algorithm. In fact, for  $\mu > d_{i,j}$ , it turns out that  $\mathcal{G}_{i,j}(\xi_i(t - \tau), \mu) = \{g_j(t - d_{i,j}, \tau)\}$  reduces to a singleton in (24) and the algorithm coincides with the S-FFCG scheme seen in the previous Subsection.

**Remark 2** - Being the general solution of problem (21) hard, it is worth to remark that, at the cost of some performance degradation, suboptimal solutions should be used. A typical choice to make to problem tractable is to fix the structure of the sets  $\Delta G_{v,i}, i = 1, \dots, N$ , e.g to boxes, spheres or ellipsoidal sets.  $\square$

**Remark 3** - It has been proved in (Casavola et al. [2009]) that the same properties pertaining to the S-FFCG scheme and reported in Proposition 2 still apply for the P-FFCG algorithm here presented. Details are here omitted for space reasons. It is finally worth remarking that low communication efforts are necessary for implementing the distributed strategies, that anyway do not affect the on-line computational burdens because data are exchanged only after the computation and application of local commands.  $\square$

## 5. ILLUSTRATIVE EXAMPLE: COORDINATION OF AUTONOMOUS VEHICLES

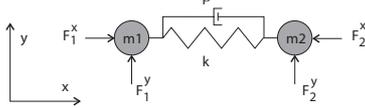


Fig. 2. Planar system of two dynamically coupled masses.

In Fig. 2, a system consisting of two dynamically coupled masses is presented. Such a system, we will refer to as  $m2$ -system, represents in essence a pair of unmanned vehicles transporting an elastic membrane. The system is described by the following equations

$$\begin{aligned} m_i \ddot{x}_i &= -k(x_i - \sum_{j \in \mathcal{N}_i} x_j) - \beta(\dot{x}_i - \sum_{j \in \mathcal{N}_i} \dot{x}_j) + F_i^x \\ m_i \ddot{y}_i &= -k(y_i - \sum_{j \in \mathcal{N}_i} y_j) - \beta(\dot{y}_i - \sum_{j \in \mathcal{N}_i} \dot{y}_j) + F_i^y \end{aligned} \quad (25)$$

where  $(x_i, y_i), i \in \mathcal{A} = \{1, 2\}$  are the coordinates of the  $i$ -th mass position w.r.t a cartesian reference and  $(F_i^x, F_i^y), i \in \mathcal{A}$ , the components along the same reference frame of the forces acting as inputs for the two slave systems.  $\mathcal{N}_i$  denotes the neighborhood of the  $i$ -th agent, in this case  $\mathcal{N}_1 = \{2\}$  and  $\mathcal{N}_2 = \{1\}$ . The following system parameters are assumed  $\beta = 1 \frac{[N \cdot sec]}{[m]}$ ,  $k = 1 \frac{[N]}{[m]}$ ,  $m_i = 1 [Kg], \forall i \in \mathcal{A}$ , and a sampling time of  $T_c = 0.1 [sec]$  is employed in the simulations. Each subsystem has been precompensated by an optimal LQ state feedback local controller.

The problem we consider here is the coordination of the planar motions of those two masses along two continuously parameterized paths  $r_1(\alpha_1) \in \mathbb{R}^2, r_2(\alpha_2) \in \mathbb{R}^2$  where  $\alpha_1 \in [0, \bar{\alpha}], \alpha_2 \in [0, \bar{\alpha}]$  are real parameters. It is required that each agent tracks its own paths by progressively increasing the value of  $\alpha_i(t)$  at the maximum speed complying with the following local and coupling constraints

$$\begin{aligned} |F_i^j(t)| &\leq 0.5 [N] & j = x, y, i \in \mathcal{A}, \\ |\nu_i(t) - r_i(\alpha_i(t))| &\leq 0.05 [m] & i \in \mathcal{A}, \\ |\alpha_i(t) - \alpha_j(t)| &\leq 0.04, & i \in \mathcal{A}, j \in \mathcal{N}_i. \end{aligned} \quad (26)$$

The first set of inequalities represents input-saturation constraints on the four forces  $F_i^x$  and  $F_i^y, i \in \mathcal{A}$ , acting as

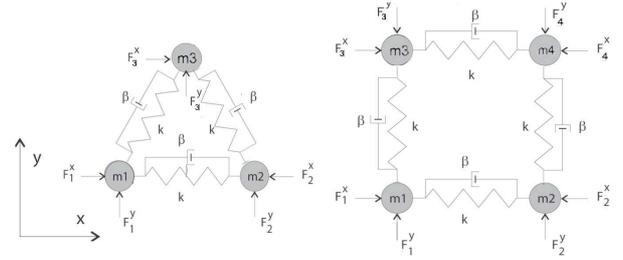


Fig. 3. a) Planar system of three dynamically coupled masses ( $m3$ -system) b) Planar system of four dynamically coupled masses ( $m4$ -system)

inputs of the vehicles. They have to be taken into account in order to avoid the generation of trajectories out of the actuator ranges. The second set of constraints represents the component-wise accuracy of the vehicle positions  $\nu_i(t) = [x_i(t), y_i(t)]$  with respect to the target motion  $r_i(\alpha_i(t))$ . Finally, the third group of constraints represents the coordination constraints between the agents: the two agents have never to be "too far" in the parameter progression in order to maintain the formation shape. In this example we will consider the rigid motions  $r_1(\alpha), r_2(\alpha)$  for  $\alpha \in [0, 4]$  reported in Figure 4.a, consisting of a combination of circular and translational uniform motions. For comparisons, in Figures 4.b -4.d, the system trajectories achieved by the use of the centralized Standard CG strategy (Gilbert et al. [1995]-Casavola et al. [2006]) and of the two presented distributed approaches are depicted for the first 1000 simulation steps. Those trajectories corresponds to the evolution of  $\alpha_i(t), i = 1, 2$  shown in Figure 5. As expected, the use of distributed schemes introduces a certain level of conservativeness w.r.t. standard centralized CG approaches. However, it is worth to note here with regard to the P-FFCG method, that even if the cartesian decomposition undertaken in this example was based on the very conservative computation of the maximal volume inscribed hypercubes, the performance degradation are very moderate. In order to evaluate the scalability of the proposed algorithms, formations of three and four masses, hereafter denoted as  $m3$ -system and  $m4$ -system, have been considered with masses on the vertices; of an equilateral triangle for  $m3$ -system (see Figure 3.a) and of a square for  $m4$ -system (see Figure 3.a), and the coupling elements on the edges. The same constraints (26) and parameterized path used for the two-masse models (shown in Figure 6.a and Figure 7.a) have been considered. Simulative results are depicted in Figures 6-7. Finally in Table I computational burdens and network usage are reported for all schemes.

## 6. CONCLUSIONS

In this paper, a novel class of distributed CG schemes has been developed for dynamically coupled linear systems subject to local and global constraints which require low-demanding communication efforts. The key point was to resort to a novel FF-CG scheme that does not require an explicit measure of the state to be implemented. Two distributed coordination algorithms have been singled out and the results on constraints fulfillment and stability highlighted. Comparisons with central solutions have been also presented and commented in the final illustrative example. The presented results are encouraging and stimulate further research on the topic.

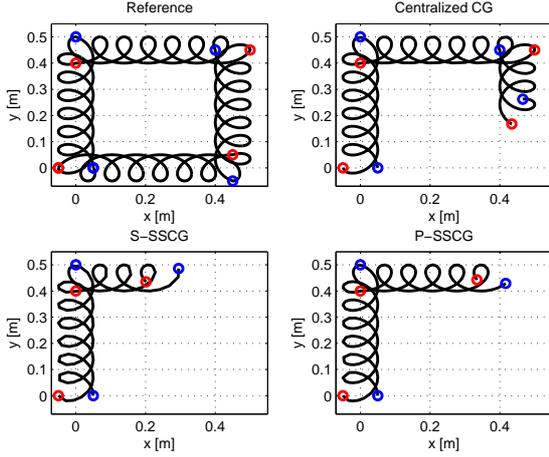


Fig. 4. Tracking trajectories for  $m2$ -system: a) Reference Trajectories, b) Standard CG, c) S-FFCG d) P-FFCG

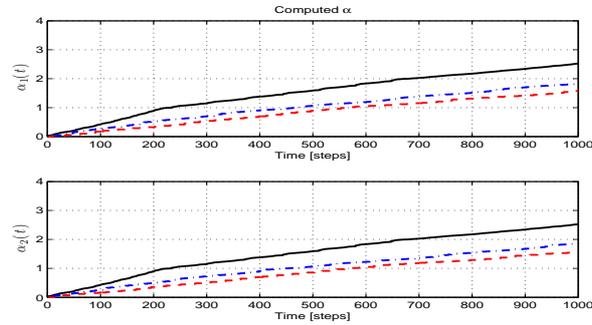


Fig. 5. Computed  $\alpha_1(t)$ ,  $\alpha_2(t)$  for  $m2$ -system: Standard CG (-), P-FFCG (- -), S-FFCG(- . -).

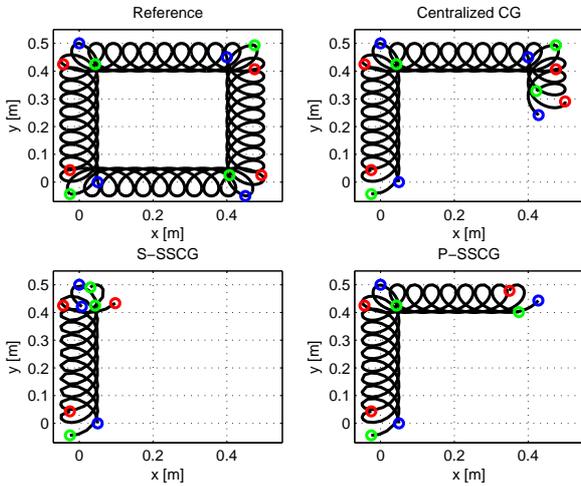


Fig. 6. Tracking trajectories for  $m3$ -system: a) Reference Trajectories, b) Standard CG, c) S-FFCG d) P-FFCG

	CPU Time [ms]			TX Data (bit/agent)		
	m2	m3	m4	m2	m3	m4
CG	5.0	5.5	6.1	384	576	768
P-FFCG	3.8	4.7	4.8	128	328	512
S-FFCG	0.47	0.38	0.35	128	192	256

Table 1. CPU time and transmitted bit per agent

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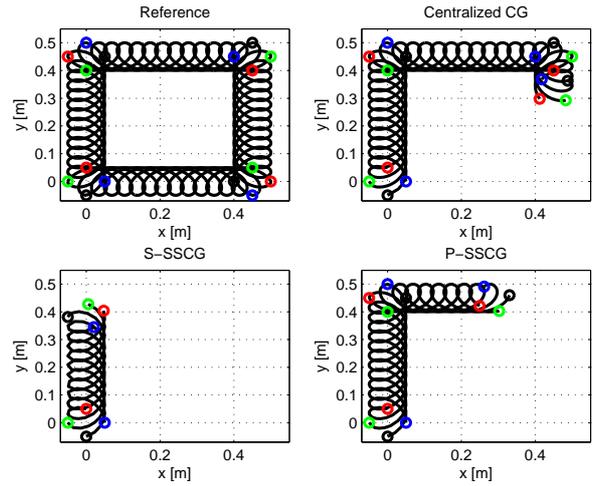


Fig. 7. Tracking trajectories for  $m4$ -system: a) Reference Trajectories, b) Standard CG, c) S-FFCG d) P-FFCG

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