A FeedForward Command Governor Strategy for Constrained Linear Systems

Emanuele Garone * **Francesco Tedesco** * **Alessandro Casavola** *

* *DEIS - Università degli Studi della Calabria, Rende(CS), 87036, ITALY,*
{casavola,egarone,ftedesco}@deis.unical.it

Abstract: This paper proposes a novel class of Command Governor (CG) strategies for input/state constrained linear systems subject to bounded disturbances in the absence of explicit state measurement or its estimation. While in traditional CG schemes, the set-point manipulation is undertaken on the basis of either the actual measure of the state or of its suitable estimation, it is shown here that the CG design problem can be solved with limited performance degradation and with similar properties also in the case that such an explicit measure is not available, by forcing the state evolutions to stay "*not too far*" from the manifold of feasible steady-states. This approach, which will be referred to as *Feed Forward CG* (FF-CG), may be a convenient alternative CG solution in all situations whereby the cost of measuring the state may be a severe limitation, e.g. in distributed or decentralized applications. In order to evaluate the method here proposed, numerical simulations on a physical plant have been performed and comparisons with the standard state-feedback CG solution reported.

Keywords: Nonlinear Control, Predictive Control, Command Governor, Sensorless Control .

1. INTRODUCTION

Command Governor strategies have recently gained interest in the control system literature for their capability to rigorously achieve the on-line fulfillment of set-membership-in-time constraints, via the modification of the prescribed reference signal, whereas allowing the off-line control design phase to be undertaken without considering their presence altogether. The CG design problem is usually solved by resorting to the theory of the *maximal output admissible set* introduced in Gilbert and Tin-Tan. [1991] and Gilbert and al. [1995]. Many mature assessments of the CG approach can be found in Gilbert and Kolmanovsky [2001]. In particular, CG schemes dealing with disturbances were considered in Gilbert and Kolmanovsky [1999] and Casavola et al. [2000], with model uncertainties in Bemporad and Mosca [1998] and Casavola et al. [2000] and with partial state information in Angeli et al. [2001]. For specific results on CG applied to nonlinear systems see e.g. Bemporad [1998], Angeli et al. [1999], Angeli and Mosca [1999] and Gilbert and Kolmanovsky [2001], for networked master/slave frameworks Casavola et al. [2006] and for recent results on hybrid Piecewise-Affine systems Falcone et al. [2009].

The Command Governor is a nonlinear device which is added to a primal compensated plant that, in the absence of it, is designed so as to exhibit stability and good tracking performance. The CG main objective is to modify, whenever necessary, the prescribed reference signal to be supplied to such a pre-compensated system if the unmodified application would lead to constraint violations. This modification is typically achieved according to a receding horizon philosophy consisting of solving on-line at each time instant a constrained convex optimization problem whose constraints take into account the future system state predictions. A system equipped with a CG takes a special simplified structure at the cost typically of performance degradation with respect to more general approaches, e.g. constrained predictive control. CG usage can be however justified in industrial applications wherein a massive amount of flops per sampling time is not allowed, and/or one is typically only commissioned to add to existing standard PID-like compensators peripheral units which, as CG's, do not change the primal compensated control system. In the above traditional contexts, the CG action is determined on the basis of the knowledge of the actual measured state.

In this paper, on the contrary, we will introduce a novel solution to the CG problem, hereafter referred to as the *FeedForward CG* (FF-CG) approach, which, at the price of some additional conservativeness, is able to accomplish the CG task in the absence of an explicit measure of the state. The idea behind such an approach is that, if sufficiently slow and smooth transitions in the reference modifications are acted by the CG unit, the state evolutions remain *not too far* from the manifold of feasible steady-states. Thus, because of the asymptotical stability of the system at hands, one can have a high confidence on the expected value of the state, even in the absence of an explicit measure, which is supposedly close to the feasible steady-state corresponding to the application of a constant set-point as a reference. Clearly, the scheme acts is a feed-forward way but the reference modification is implicitly undertaken on the basis of the *expected* value of the current state *inferred* by the knowledge of the currently applied constant set-point.

A preliminary version of this strategy has been proposed in Garone et al. [2009] where it was instrumental for the design of distributed CG strategies for large scale networked applications. Here, the analysis is extended further on by considering the presence of bounded disturbances that might deteriorate the tracking performance. Moreover, a tighter characterization of the feasible sets underlying the CG action computation is here also presented. This allows the achievement of improved tracking performance with respect to the solution proposed in Garone et al. [2009]. The paper is organized as follows. In Section II the CG problem is defined and some preliminary aspects on the idea of a sensorless CG algorithm are discussed. In Section III the Steady State CG scheme is introduced and some implementation details are given for particular cases. In Section IV numerical simulations are reported in order to show the proposed algorithm's effectiveness. Some conclusions end the paper.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Let consider the closed-loop system described by the following discrete-time model

$$\begin{cases} x(t+1) &= \Phi x(t) + Gg(t) + G_d d(t) \\ y(t) &= H_y x(t) \\ c(t) &= H_c x(t) + Lg(t) + L_d d(t) \end{cases} \tag{1}$$

where: $t \in \mathbb{Z}$, $x \in \mathbb{R}^n$ is the state vector (which includes the controller states under dynamic regulation), $g \in \mathbb{R}^{m_i}$ is the manipulable reference vector which, if no constraints (and no CG) were present, would coincide with the desired reference $r \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$ is the output vector which is required to track r. The vector $d \in \mathbb{R}^{n^d}$ is a disturbance signal assumed to belong to the closed ball $\mathcal{D} := \{d : \|d\|_2 \leq d_{max}\}$. Finally, $c \in \mathbb{R}^{n^c}$ represents the constrained vector which has to fulfill the set-membership constraint

$$c(t) \in \mathcal{C},\ \forall t \in \mathbb{Z}_+, \tag{2}$$

$\mathcal{C}$ being a polyhedral set. It is further assumed that:

A1. The overall system (1) is asymptotically stable.

A2. System (1) is off-set free, i.e. $H_y(I_n - \Phi)^{-1}G = I_m$.

Roughly speaking, the CG design problem we want to solve is that of determining, at each time step t, a suitable reference signal $g(t)$ which is the best approximation of $r(t)$ such that its application never produces constraints violation, i.e. $c(t) \in \mathcal{C}, \forall t \in \mathbb{Z}_+$. Classical solutions of the above stated CG design problem (see Bemporad et al. [1997], Casavola et al. [2000, 2006]) have been achieved by finding, at each time t, a CG action $g(t)$ as a function of the current reference $r(t)$ and measured state $x(t)$

$$g(t) := \underline{g}(r(t), x(t)) \tag{3}$$

such that $g(t)$ is the best approximation of $r(t)$ under the condition $c(t) \in \mathcal{C}$. Here we will focus on a slight different approach to the CG design problem in which no measure of the state vector is required to determine $g(t)$. Such an approach, hereafter referred to as *Feed-Forward* CG (FF-CG), will be described in next sections. In order to better introduce the key ideas, let's consider temporarily the disturbance-free ($d(t) \equiv 0_{n_d}$) case and adopt the following notations for the steady-state solutions of (1) to a constant command $g(t) = g$

$$x_g := (I_n - \Phi)^{-1}Gg,\ y_g := H_y(I_n - \Phi)^{-1}Gg,\ c_g := H_c x_g + Lg. \tag{4}$$

The idea implicitly employed in this approach is that, if the manipulable reference signal $g(\cdot)$ is selected to be "slow enough" w.r.t. system dynamics, then, because of system stability (see **A1**), the constrained vector $c(t)$ can be maintained within a certain known (and "small") distance $\rho(t) > 0$ from the closed-loop steady-state equilibrium $c_{g(t)}$, which results from the application of the constant set-point $g(t)$ for a sufficient number of steps, i.e.

$$c(t) - c_{g(t-1)} \in \mathcal{B}_{\rho(t)}. \tag{5}$$

where $\mathcal{B}_{\rho(t)}$ represents the ball of radius $\rho(t)$ centered at the origin, with $\rho(t)$ suitably chosen on-line, at each time instant, in such a way to ensure constraints fulfilment. As will be highlighted in the sequel, because of **A1**, (5) induces a similar boundedness condition on $x(t) - x_{g(t-1)}$ under certain conditions. The above discussion allows us to conceive CG schemes in which, instead of considering the dependence on the measured state $x(t)$ as shown in (3), decisions at time t are undertaken on the basis of the condition (5) and, hence, on the basis of pairs $\left(c_{g(t-1)}, \rho(t-1)\right)$. Moreover, because $c_{g(t-1)}$ univocally depends on the command signal applied at the previous time step $g(t-1)$, we can finally reformulate the Feed-Forward CG problem as the one of finding a command signal $g(t)$ as an explicit function of

$$g(t) = \underline{g}(r(t), g(t-1), \rho(t)) \tag{6}$$

where $g(t)$ is the best approximation of $r(t)$ to be computed so as to ensure constraints satisfaction along the system virtual evolutions.

3. THE STANDARD AND THE FEED-FORWARD COMMAND GOVERNOR APPROACHES

In order to make precise statements and comparisons with existing techniques, we describe first the classic CG approach which explicitly makes use of the measure of the state. Consider the constrained closed-loop system (1)-(2) satisfying assumptions **A1-A2**. Let the state be measurable at each time instant and consider the CG design problem as formulated in (3). In order to simplify the developments, let us exploit the linearity to separate the effects of initial conditions and commands from those of disturbances, i.e.

$$x(t) = \hat{x}(t) + \tilde{x}(t),\ c(t) = \hat{c}(t) + \tilde{c}(t),\ y(t) = \hat{y}(t) + \tilde{y}(t) \tag{7}$$

where $\hat{x}(t)$ (and the same for $\hat{c}(t)$ and $\hat{y}(t)$) is the disturbance-free component of the state (depending only on the initial state condition $x(0)$ and commands) whereas $\tilde{x}(t)$ depends only on the disturbances (starting from zero initial conditions). Next, consider the following set recursion

$$\begin{aligned} \mathcal{C}_0 &:= \mathcal{C} \sim L_d\mathcal{D} \\ \mathcal{C}_k &:= \mathcal{C}_{k-1} \sim H_c\Phi^{k-1}G_d\mathcal{D} \\ \mathcal{C}_\infty &:= \bigcap_{k=0}^{\infty} \mathcal{C}_k \end{aligned} \tag{8}$$

where for given sets $\mathcal{A}, \mathcal{E} \subset \mathbb{R}^n$, $\mathcal{A} \sim \mathcal{E}$ is the Pontryagin set difference defined as $\mathcal{A} \sim \mathcal{E} := \{a : a + e \in \mathcal{A}, \forall e \in \mathcal{E}\}$. It can been shown that the sets $\mathcal{C}_k$, if non-empty, are convex because of the convexity of $\mathcal{C}$.

Let us introduce the set-valued future predictions (*virtual evolutions*) of the c-variable along the *virtual time* k under a constant *virtual command* $g(k) \equiv g$ and for all the possible disturbance sequence realizations $\{d(l) \in \mathcal{D}\}_{l=0}^{k}$ from initial state x (at virtual time $k = 0$)

$$c(k,x,g,d(\cdot)) = \bigcup_{d(\cdot)\in\mathcal{D}} \left\{ H_c \left(\Phi^k x + \right.\right.$$
$$\left.\left. + \sum_{i=0}^{k-1} \Phi^{k-i-1}(Gg + G_d d(i)) \right) + Lg + L_d d(k) \right\} \tag{9}$$

The latter can be rewritten as the sum of two terms:

$$c(k,x,g,d(\cdot)) = \overline{c}(k,x,g) + \tilde{c}(k,d(\cdot)) \tag{10}$$

where $\overline{c}(k,x,g)$ represents the disturbance-free evolution of the c-variable along the *virtual time* k under a constant *virtual command* $g(k) \equiv g$ and initial state x and $\tilde{c}(k,d(\cdot))$ is the set-valued virtual evolutions of the c-variable due to all possible disturbance sequence realization $\{d(l) \in \mathcal{D}\}_{l=0}^{k}$. It is possible to prove that

$$\begin{array}{c}\overline{c}(k,x,g) \in \mathcal{C}_k,\ \forall k \in \mathbb{Z}_+ \\ \Downarrow \\ c(k,x,g) = \overline{c}(k,x,g) + \tilde{c}(k,d(\cdot)) \subset \mathcal{C},\ \forall k \in \mathbb{Z}_+\end{array} \tag{11}$$

Thus, the constraints fulfilment can be ensured by only considering the disturbance-free evolutions of the system (1) and one can adopt a "worst-case" approach. To this end, let us introduce, for a given sufficiently small scalar $\delta > 0$, the sets:

$$\begin{array}{ll}\mathcal{C}^\delta & := \mathcal{C}_\infty \sim \mathcal{B}_\delta \\ \mathcal{W}^\delta & := \{g \in \mathbb{R}^m :\ c_g \in \mathcal{C}^\delta\}\end{array} \tag{12}$$

where $\mathcal{B}_\delta$ is the ball of radius δ centered at the origin and $\mathcal{W}^\delta$, which we will assume non-empty, the set of all constant commands g whose corresponding disturbance-free equilibrium points $\hat{c}_g$ satisfy the constraints with margin δ. From the foregoing definitions and assumptions, it follows that $\mathcal{W}^\delta$ is closed and convex. Then, by exploiting recursions (8) we can define, for any given state x, the set $\mathcal{V}(x)$ as

$$\mathcal{V}(x) = \{g \in \mathcal{W}^\delta :\ \overline{c}(k,x,g) \in \mathcal{C}_k,\ \forall k \in \mathbb{Z}_+\} \tag{13}$$

As a consequence, $\mathcal{V}(x) \subset \mathcal{W}^\delta$ represents, if non-empty, the set of all constant commands in $\mathcal{W}^\delta$ whose virtual c-evolutions starting from x at virtual time $k = 0$ satisfy the constraints also during transients. Finally, the standard CG design problem can be solved at each time t by adopting the selection algorithm

The standard CG Algorithm

REPEAT AT EACH TIME t

1.1 SOLVE
$$g(t) = \arg \min_{g\in\mathcal{V}(x(t))} \parallel g - r(t) \parallel_\Psi^2 \tag{14}$$

1.2 APPLY $g(t)$

3.1 The proposed FF-CG approach

Here the goal is to present a different approach to the CG problem that enables us to deal with the case that no state measurements are available to the supervisory unit. The idea underlying such an approach consists of ensuring that any admissible variation of the manipulated reference $g(\cdot)$ always produces a guaranteed bounded perturbation on the actual constrained vector c around a suitable feasible steady-state value. Such a property can be ensured by opportunely limiting the reference variations by means of the following technical expedients:

(1) the computation of a new FF-CG action $g(t)$ is performed every τ steps, being τ a suitable integer to be determined, rather than at each time instant t as in the standard CG approach. Moreover, each new FF-CG command is applied for exactly τ steps;

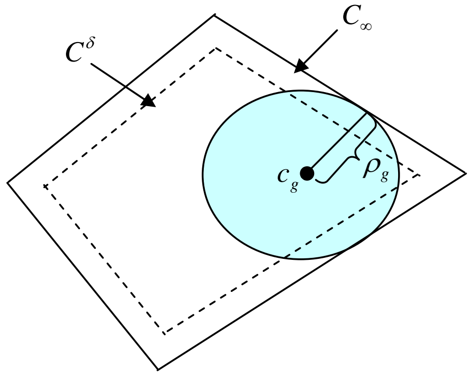

Fig. 1. Geometrical representation of condition (21) for $c \in \mathbb{R}^2$

(2) the displacement between the new FF-CG command $g(t)$ and the previous one $g(t-\tau)$ is explicitly bounded during the FF-CG computation, i.e.
$$g(t) - g(t-\tau) \in \Delta\mathcal{G}(g(t-\tau), \rho(t-\tau)), \tag{15}$$
where the integer $\tau > 0$ and the closed and convex set $\Delta\mathcal{G}(g,\rho) \subset \mathbb{R}^m$ are determined from the outset whereas $\rho(t)$ is a time-varying scalar parameter selected on-line. The definitions and usage of τ, $\Delta\mathcal{G}(g,\rho)$ and $\rho(\cdot)$ will be introduced later.

The first difference with the standard CG design method is in the way the effect of the disturbances is taken into account. In fact, while in the standard CG approach the *virtual evolutions* $\overline{c}(k,x,g)$ are computed on the basis of the measured state $x(t)$ and the effect of disturbances on the predictions can be suitably characterized along the prediction horizon, in the FF-CG approach, because the measure of the actual state is assumed not directly available, all past realizations of the disturbances have to be taken into account as if they were acting on each element of the sequence $\overline{c}(k,x,g)$ from an arbitrarily long time in the past. Specifically, by considering the notations introduced in (7), the set-valued virtual evolution of the c variable along the *virtual time* k under a constant *virtual command* $g(k) \equiv g$ and for initial time x (at $k = 0$) can be rewritten as the sum of three terms

$$c(k,x,g,d(\cdot)) = \overline{c}(k,\hat{x},g) + \tilde{c}(k,d(\cdot)) + H_c\Phi^k\tilde{x}. \tag{16}$$

Note that if no measure of x is available, $\tilde{x}$ is not known punctually but is a set that depends on all possible past values of the disturbance $d(\cdot) \in \mathcal{D}$. Then, it results

$$H_c\Phi^k\tilde{x} = \bigcup_{d(\cdot)\in\mathcal{D}} \left\{ H_c\Phi^k \sum_{i=-\infty}^{0} H_c\Phi^{-i}G_d d(i-1) \right\} \subseteq \sum_{i=k}^{\infty} H_c\Phi^i G_d\mathcal{D} \tag{17}$$

and

$$\overline{c}(k,\hat{x},g) \in \mathcal{C}_\infty,\ \forall k \in \mathbb{Z}_+ \Longrightarrow c(k,x,g,d(\cdot)) \subset \mathcal{C},\ \forall k \in \mathbb{Z}_+ \tag{18}$$

As a consequence, in the present context, the constraints fulfilment is obtained by ensuring that

$$\overline{c}(k,\hat{x},g) \in \mathcal{C}_\infty, \forall k \in \mathbb{Z}_+ \tag{19}$$

Such a condition can be further simplified by manipulating the *virtual evolutions* $\overline{c}(k,x,g)$ as follows

$$\overline{c}(k,\hat{x},g) = c_g + H_c\Phi^k(\hat{x} - x_g) \tag{20}$$

where c_g is its steady-state component and $H_c\Phi^k(\hat{x} - x_g)$ the transient evolution. Like in the standard CG solution, we will restrict our attention to virtual commands g within the set $\mathcal{W}^\delta$. Then, the steady-state component of the virtual evolutions will always belong to $\mathcal{C}^\delta$. As depicted in Fig. 1, a sufficient condition to ensure that the constraints will be satisfied, although in a quite arbitrary and conservative way, is that of ensuring that the vanishing transient component of $\bar{c}$ is confined into a ball of radius ρ_g, viz.

$$\|H_c\Phi^k(\hat{x}-x_g)\| \le \rho_g, \forall k \ge 0 \tag{21}$$

where ρ_g represents the minimum distance between c_g and the border of $\mathcal{C}_\infty$. Such a quantity can be formally defined as the solution of the following optimization problem

$$\begin{aligned} \rho_g &:= \arg\max_\rho \rho \\ \text{subject to } & \mathcal{B}_\rho(c_g) \subseteq \mathcal{C}_\infty \end{aligned} \tag{22}$$

where $\mathcal{B}_\rho(c_g)$ represents the ball of radius ρ centered in c_g. Please note that, by construction, $\rho_g \ge \delta, \forall g \in \mathcal{W}^\delta$. Then, the key idea behind the construction of an effective FF-CG algorithm is as follows: let's decide to modify the FF-CG command signal only every τ steps and assume that, at time $t-\tau$, a command $g(t-\tau) \in \mathcal{W}^\delta$ has been computed such that the transient component of $\bar{c}(k, \hat{x}(t-\tau), g(t-\tau))$ is confined in a ball of known radius $\rho(t-\tau)$, that is also contained into the ball of radius $\rho_{g(t-\tau)}$, which makes the condition (21) to hold true, i.e.

$$\| H_c\Phi^k(\hat{x}(t-\tau) - x_{g(t-\tau)}) \| \le \rho(t-\tau) \le \rho_{g(t-\tau)}, \ \forall k \ge 0. \tag{23}$$

If we build an algorithm that, on the basis of the above information, is able to select at time t a new command $g(t) \in \mathcal{W}^\delta$ and a scalar $\rho(t) \ge 0$ such that the transient component of $\bar{c}(k, \hat{x}(t), g(t))$ is confined within a ball of radius $\rho(t) \le \rho_{g(t)}$, then the constraints would be again satisfied, i.e.

$$\| H_c\Phi^k(\hat{x}(t) - x_{g(t)}) \| \le \rho(t) \le \rho_{g(t)}, \forall k \ge 0 \tag{24}$$

and, by mathematical induction, FF-CG commands $g(t)$ would be proved to exist at each time instants $t \in \mathbb{Z}_+$ provided that it would exist at time $t=0$. To this end, let's concentrate on how the term $\| H_c\Phi^k(\hat{x}(t) - x_{g(t)}) \|$ can be bounded. The key observation here is that, if we wait for a sufficient long time after the application of a new FF-CG command, the transient contribution decreases and can be bounded within a certain percentage of its initial bound. More formally, let us introduce the following notion of *Generalized Settling Time*:

Definition *(Generalized Settling Time)* - The integer $\tau > 0$ is said to be a *Generalized Settling Time with parameter* γ, with $0 < \gamma < 1$, for the pair (H_c, Φ), if

$$\begin{gathered} \|H_c\Phi^k x\| \le M(x), \quad \forall k = 0,1,...,\tau-1 \\ \Downarrow \\ \|H_c\Phi^{\tau+k}x\| \le \gamma M(x), \forall k \ge 0 \end{gathered} \tag{25}$$

holds true for each $x \in \mathbb{R}^n$, with the real $M(x) > 0$ any upper-bound to $\|H_c\Phi^k x\|, \forall k \ge 0$. □

As a consequence, if the time interval between two command variations τ were chosen to be a generalized settling time with parameter $\gamma \in (0,1)$ and if $g(t) = g(t-\tau)$ were selected at time t as solution of the FF-CG design problem, the disturbance free c-transient would be bounded as follows

$$\| H_c\Phi^k(\hat{x}(t) - x_{g(t-\tau)}) \| \le \gamma\rho(t-\tau), \quad \forall k \ge 0, \tag{26}$$

because

$$\Phi^\tau(\hat{x}(t-\tau) - x_{g(t-\tau)}) = (\hat{x}(t) - x_{g(t-\tau)}). \tag{27}$$

Note that, obviously, the choice $g(t) = g(t-\tau)$ would comply with constraints (24) because the new bound on the c-transient $\rho(t) = \gamma\rho(t-\tau)$ would ensure $\rho(t) \le \rho_{g(t-\tau)}$.

The latter observation can be used to characterize the set of all possible feasible commands complying with the constraint (24). To this end, let's parametrize the generic, non necessarily feasible, command $g \in \mathcal{W}^\delta$ to be applied to the system at time t as the sum of the previously applied command $g(t-\tau)$ and of a command increment Δg to be determined by the FF-CG, i.e. $g = g(t-\tau) + \Delta g$. By noticing that $x_{\Delta g} = x_g - x_{g(t-\tau)}$ and by exploiting the triangular inequality, we can easily bound the transient component of $\bar{c}(k, \hat{x}(t), g)$ as follows

$$\begin{aligned} \|H_c\Phi^k(\hat{x}(t) - x_g)\| &= \|H_c\Phi^k(\hat{x}(t) - x_{g(t-\tau)}) - H_c\Phi^k x_{\Delta g}\| && (28) \\ &\le \|H_c\Phi^k(\hat{x}(t) - x_{g(t-\tau)})\| + \|H_c\Phi^k x_{\Delta g}\| && (29) \\ &\le \gamma\rho(t-\tau) + \|H_c\Phi^k x_{\Delta g}\| && (30) \\ &\le \rho(t), \ k \ge 0. && (31) \end{aligned}$$

Then, a possible way to define the scalar $\rho(t)$ at time t complying with (31) is:

$$\rho(t) := \gamma\rho(t-\tau) + max_{k\ge 0} \| H_c\Phi^k x_{\Delta g} \| \tag{32}$$

where such a bound, as will be clearer in the next Section, is finitely determinable. Moreover, note that it does not depend on $\hat{x}(t)$, assumed in fact not available here, but only on $\rho(t-\tau) \le \rho_{g(t-\tau)}$ and on the free command increment Δg, which has to be determined so that (24) holds true. By direct examination, the latter requirement simply consists of selecting Δg such that $\rho(t) \le \rho_g$. In fact, the last condition can be explicitly rewritten as follows

$$\| H_c\Phi^k x_{\Delta g} \| \le \rho_{g(t-\tau)+\Delta g} - \gamma\rho(t-\tau), \forall k \ge 0 \tag{33}$$

and, by recalling the static map $x_{\Delta g} = (I-\Phi)^{-1}G\Delta g$, the set of all feasible FF-CG commands at time t can finally be characterized as

$$\begin{cases} g \in \mathcal{W}^\delta \\ (g - g(t-\tau)) \in \Delta\mathcal{G}(g(t-\tau), \rho(t-\tau)) \end{cases} \tag{34}$$

where $\Delta\mathcal{G}(g,\rho)$ is defined as the set of all τ-step incremental commands from $g(t-\tau)$ ensuring inequality (33) to hold true:

$$\Delta\mathcal{G}(g,\rho) = \left\{\Delta g : \|H_c\Phi^k(I-\Phi)^{-1}G\Delta g\| \le \rho_{g+\Delta g} - \gamma\rho, \forall k \in \mathbb{Z}_+\right\}. \tag{35}$$

This set can be proved to be convex, closed and non-empty (please refer to Casavola et al. [2009] for details). Then, a quadratic selection index as the one in (36) can be used on-line to determine at each time instant the best approximation of $r(t)$ complying with the prescribed constraints.

The FF-CG Algorithm

REPEAT AT EACH TIME $t = k\tau, k = 1, 2 \ldots$

1.1 SOLVE
$$g(t) = \arg\min_g \| g - r(t) \|^2_\Psi \tag{36}$$
SUBJECT TO (34)

1.2 APPLY $g(t)$ FOR THE NEXT τ STEPS

1.3 UPDATE $\rho(t) = \gamma\rho(t-\tau) + max_{k\ge 0} \| H_c\Phi^k(I-\Phi)^{-1}G\Delta g(t) \|$.

The above FF-CG scheme enjoys the following properties whose proof is here omitted for space limitation (please refer to Casavola et al. [2009] for details).

Theorem *1.* - Let assumptions **A1-A2** be fulfilled. Consider system (1) along with the **FF-CG** selection rule and let an admissible command signal $g(0) \in \mathcal{W}^\delta$ be applied at $t = 0$, with scalars $\rho(0)$ and $\rho_{g(0)}$ existing such that

$$\| H_c \Phi^k (x(0) - x_{g(0)}) \| \leq \rho(0) \leq \rho_{g(0)}, \ \forall k \geq 0 \quad (37)$$

Then:

(1) At each decision time $t = \kappa\tau, \kappa \in \mathbb{Z}_+$, the minimizer in (36) uniquely exists and can be obtained by solving a convex constrained optimization problem;

(2) The system acted by the FF-CG never violates the constraints, i.e. $c(t) \in \mathcal{C}$ for all $t \in \mathbb{Z}_+$ regardless of any possible admissible disturbance realization $d(\cdot) \in \mathcal{D}$;

(3) At each time instant $t \in \mathbb{Z}_+$, the disturbance-free part of constrained vector $c(t)$ is known to lie in a ball of center $c_{g(t)}$ and radius $\rho(t)$:

$$\hat{c}(t) \in \left(c_{g(t)} + \mathcal{B}_{\rho(t)} \right), \forall t \in \mathbb{Z}_+ \quad (38)$$

where $g(t) = g\left(\left\lfloor \frac{t}{\tau} \right\rfloor \tau\right)$ and $\rho(t) = \rho\left(\left\lfloor \frac{t}{\tau} \right\rfloor \tau\right)$, with $\lfloor \cdot \rfloor$ denoting the standard floor operator. As a consequence, the constraint vector $c(t)$ will satisfies

$$c(t) \in \left(c_{g(t)} + \mathcal{B}_{\rho(t)} \right) + \Delta_\infty, \forall t \in \mathbb{Z}_+ \quad (39)$$

(4) Whenever $r(t) \equiv r$, with r a constant set-point, the sequence of $g(t)$ converges in finite time either to r or to its best admissible steady-state approximation $\hat{r}$

$$\exists t' > 0 \, t.c. \ g(t) = \hat{r} := \arg\min_{g \in \mathcal{W}^\delta} \| g - r \|^2_\Psi, \forall t \geq t' \quad (40)$$

Moreover, it results

$$\lim_{t \to \infty} \hat{x}(t) = x_{\hat{r}}, \ \lim_{t \to \infty} \hat{y}(t) = y_{\hat{r}} = \hat{r}, \ \lim_{t \to \infty} \hat{c}(t) = c_{\hat{r}} \quad (41)$$

□

Remark 1 - It is worth pointing out that the determination of τ as a generalized settling time with parameter γ can be performed off-line. Moreover, we want also to remark that the characterizations of both $\Delta\mathcal{G}(g, \rho)$ defined in (35) and the computation of the updating strategy for $\rho(t)$ in (32) are finitely determinable operations with respect to k. In fact, thanks to the definition of generalized settling time and for the asymptotical stability of the system, it can be shown the verification of the conditions underlying (35) and (32) for $k = 0, 1, ..., \tau$ is equivalent to verify them for all $k \geq 0$. Computational details for the determination of τ, $\Delta\mathcal{G}(g, \rho)$ and $\rho(t)$ may be found in (Casavola et al. [2009]). □

Remark 2 - In many practical applications input-saturations and state-related constraints may be represented by box constraints. Then a way to reduce the conservativeness is of rewriting the norm constraints with the component-wise inequalities. In next section this approach will be referred as FF-CG(Box) and from the simulation results it seems to effectively work.

4. ILLUSTRATIVE EXAMPLE: POSITION SERVOMECHANISM

The proposed FF-CG scheme is applied the position servomechanism schematically described in Figure 2. This consists of a DC-motor, a gear-box, an elastic shaft and an uncertain load. No disturbances are considered for simplicity. Technical specifications involve bounds on the shaft torsional torque T as well as on the input voltage V. System parameters are reported in Table 1.

Let θ_M and θ_L denote respectively the motor and the load angle and let

$$x_p = \left[\theta_L \ \dot{\theta}_L \ \theta_M \ \dot{\theta}_M \right]'$$

be a suitable state vector. Then, the plant can be described by the following state-space model

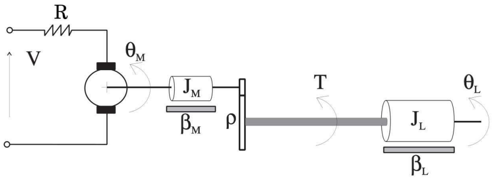

Fig. 2. Servomechanism model

Table 1. Model parameters

Symbol	Value (MKS)	Meaning
Symbol	Value (MKS)	Meaning
L_S	1.0	shaft length
d_S	0.02	shaft diameter
J_S	negligible	shaft inertia
J_M	0.5	motor inertia
β_M	0.1	motor viscous friction coefficient
R	20	resistance of armature
K_T	10	motor constant
ρ	20	gear ratio
k_θ	1280.2	torsional rigidity
J_L	$20J_M$	load inertia
β_L	25	load viscous friction coefficient
T_s	0.1	sampling time

$$\begin{cases} \dot{x}_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_\theta}{J_L} & -\frac{\beta_L}{J_L} & \frac{k_\theta}{\rho J_L} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_\theta}{\rho J_M} & 0 & -\frac{k_\theta}{\rho^2 J_M} & -\frac{\beta_M + k_T^2/R}{J_M} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_T}{R J_M} \end{bmatrix} V \\ \theta_L = \left[1 \ 0 \ 0 \ 0 \right] x_p, \ T = \left[k_\theta \ 0 \ -\frac{k_\theta}{\rho} \ 0 \right] x_p \end{cases}$$

Because the steel shaft has a finite shear strength, a maximum admissible shaft $\tau_{adm} = 50 N/mm^2$ imposes the constraint $|T| \leq 78.5398 \ Nm$ on the torsional torque. Moreover, the input DC voltage V has to be constrained within the range $|V| \leq 220 \ V$. The model is transformed in discrete time by sampling every $T_s = 0.1 s$ and using a zero-order holder on the input voltage. A digital LQR controller is used as the primal controller.

The resulting closed-loop system, when not governed by a CG unit, exhibits a very fast response but inadmissible voltage inputs and torsional torques for the references of interest, as shown in Figure. 3 for a square-wave set-point with amplitude equal to $r = 60 \ deg$ (solid line) and increasing frequency. The FF-CG unit is then applied in order to fulfill torque and voltage constraints.

The simulations for the CG governed system are reported in Figure 4, where the output $y(t)$ (upper) and the computed CG commands $g(t)$ are depicted for the same set-point of Figure 3. In this figure, the classical CG solution (CG in the figures) and both FF-CG and FF-CG(Box) methods are applied, the latter being synthesized on the basis of a generalized settling time $\tau = 7$ with contraction factor $\gamma = 0.6$. It is fair commenting that both the CG and FF-CG(Box) algorithms are built so as to exploit the fact that the system is subject to box constraints whereas the FF-CG method doesn't enjoy this capability and it expected to behave more conservatively. Interestingly enough, the performance of the three algorithms are very similar. In Figure 5, the constrained variables are depicted: in this case the input voltage and the torsional torque always satisfy the respective constraints. It is worth noticing that the trajectories of the system acted by standard CG and FF-CG (Box) are very close each other and both are much closer to the constraint boundaries than the ones resulting under the FF-CG scheme. Finally, in

Table 2 the on-line computational burdens per step of all schemes are reported.

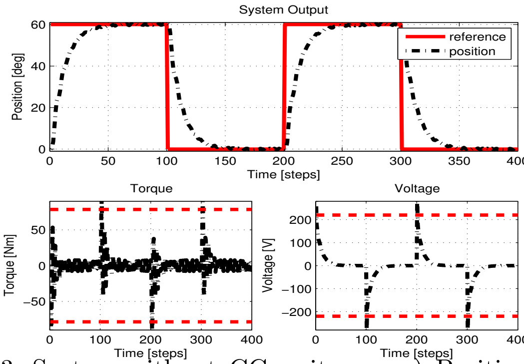

Fig. 3. System without CG unit: upper) Position lower) Constrained variables: Torsional Torque (left), voltage (right)

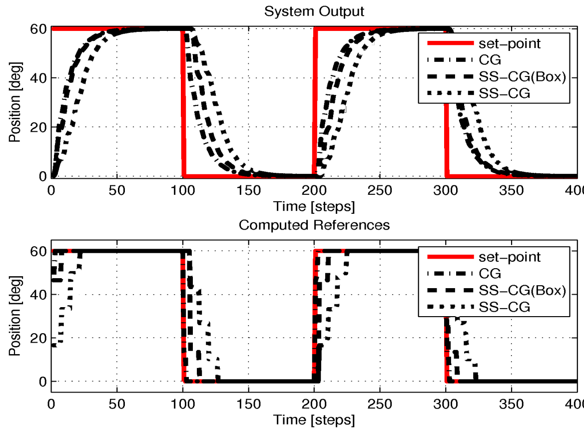

Fig. 4. System with CG unit, upper) System Output, lower) Computed CG actions.

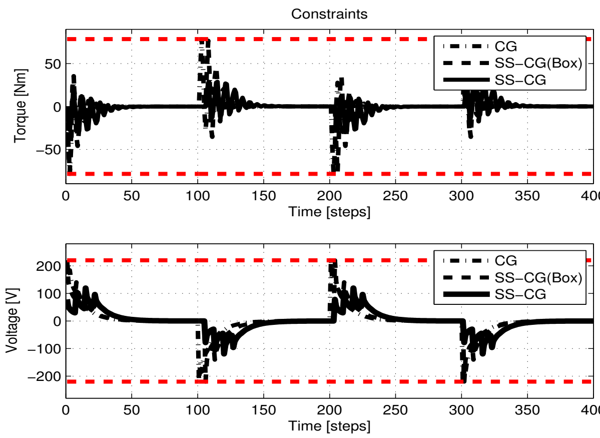

Fig. 5. Constrained variables. upper) Torque, lower) Voltage.

5. CONCLUSIONS

In this paper, a novel CG scheme is proposed which needs not be based on the explicit measure of the state to govern the set-point changes. The main idea is to limit reference variations in order to always maintain the state trajectory "not too far" from the manifold of steady-state equilibria. The properties of the proposed algorithm have been carefully studied and the differences with standard CG approaches pointed out. Comparisons with the classical CG solutions and a previously proposed

	CPU Time per step [ms]
CG	40
FF-CG(Box)	37.2
FF-CG	69.5

Table 2. On-line phase - CPU time per step

FF-CG solution have also been presented and commented in the final illustrative example.

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