An Off-Line MPC Scheme for Discrete-Time Linear Parameter Varying Systems

Alessandro Casavola, Domenico Famularo, Giuseppe Franzè and Emanuele Garone

Abstract— **An off-line Model Predictive Control (MPC) method based on ellipsoidal calculus and viability theory is described in order to address feedback regulation problems for input-saturated multi-model Linear Parameter Varying (LPV) systems. In order to reduce the computational burdens and the conservativeness of traditional robust MPC schemes, the proposed approach makes use of a scheduling linear control law which is based on a tighter refinement of the state trajectories tube. The control inputs are on-line obtained as the solutions of simple and numerically low-demanding optimization problems subject to set-membership constraints. The measurable plant parameters are directly used for feedback purposes. A numerical example is presented in order to show the benefits of the proposed regulation strategy and contrasted with a robust paradigm (the LPV hypothesis is not taken into account) and with a fast MPC algorithm for LPV systems which exploits a more traditional and conservative refinement of the state trajectory tube.**

I. INTRODUCTION

Linear Parameter-Varying (LPV) systems provide a modeling paradigm that goes substantially beyond the classical framework of linear time-invariant models. In particular, phenomena which require to capture time-varying mechanisms can be well represented within the LPV system description. The concept is even sufficiently powerful to model nonlinear systems which show fast movements between different operating regimes. Because of its advantages, recent years have witnessed the development of advanced controller synthesis algorithms for LPV systems which have sprang from robust and optimal control techniques and have some favorable features if compared with the available methods from nonlinear control.

Model Predictive Control (MPC) is a popular control technique especially well suited for dealing with constraints. Its control paradigm consists in the computation of a sequence of control actions that optimize the future evolution of the system over a given period of time. The first control move is applied to the plant and the optimization procedure is repeated again next time instant from the new reached state. A well known drawback is its inherent conservativeness, which translates into poor regulated plant performance, and heavy computational burdens. These drawbacks are especially evident in robust and scheduling MPC schemes which explicitly take into account the model plant uncertainties, time-varying plant parameters and persistent disturbances (see e.g. [11], [16]).

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In literature, proposed remedies make use of *closed-loop* predictions [13] instead of early approaches based on *open-loop* predictions, which give rise to reasonable results only under nominal conditions. In fact, the effect of disturbances or model impairments is over-estimated in methods which use open-loop predictions and the corresponding performance are poor. By now, almost all recent robust MPC schemes hold these features, the list includes methods in which the minimization is carried out over control policies which are updated on-line e.g. [9], [17].

It is generally recognized that the computation of the on-line parts of traditional robust MPC schemes is a numerically prohibitive task in many practical situations. The problem is especially severe for uncertain multi-model, linear time-varying, LPV and nonlinear schemes. Most of current research on MPC is in fact devoted to reduce such a high computational burden while still ensuring the same level of control performance. Examples of these new algorithms include explicit MPC [4] and off-line MPC paradigms based on suitable approximations of exact controllable sets [12], [8] and [6].

Although based on different ideas, all of the above approaches share the common feature to move off-line as much computations as possible. By focusing on ellipsoidal schemes, in [17] a bank of nested ellipsoids and corresponding state-feedback gains with increasingly larger performance is computed from the outset. On line, the smallest ellipsoid containing the currently measured state is determined at each time instant and the corresponding state-feedback gain put into the loop. On the same philosophy are two works, the first by [8] which is a direct extension of [17], where the worst-case optimization is substituted by the minimization of a nominal quadratic index penalizing the evolutions of a single system within the uncertain polytopic family. The second is by [1] where a control strategy for constrained multi-model uncertain plants is based on a bank of off-line pre-computed ellipsoidal sets, on-line exploited by means of a numerically low demanding optimization problem subject to a set-membership constraint.

Moving from the above considerations, we extend here the fast MPC strategy proposed in [1] for uncertain multi-model systems to the LPV plant setup which exploits the measure of the available system parameters to implement the control action in the form of a scheduling controller and ensures bounded closed-loop trajectories in the presence of persistent disturbances. A substantial reduction of conservativeness and

ISBN 978-963-311-369-1

numerical burdens is also achieved by jointly combining the scheduling control law with tighter refinement of the state trajectories tube. By means of simple arguments regarding the structure of scheduling parameters it is shown that it is possible to significantly reduce the size of the convex hulls containing the tubes of state trajectories from that obtained in [7].

A numerical example is presented in order to prove the benefits of the proposed MPC strategy in terms of attraction regions and control performance by choosing two different classes of strategies: minimum-time and minimum-energy. The obtained results are then compared with a robust paradigm (the LPV hypothesis not taken into account) and with a LPV algorithm which exploits less tight refinements of the state trajectory tube.

II. Problem formulation

Consider the following multi-model family of discrete-time linear systems

$$x(t+1) = A(p(t))x(t) + B(p(t))u(t) + G_d\, d(t) \qquad (1)$$

where $t \in \mathbb{Z}_+$; $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ the control input, $d(t) \in \mathbb{R}^{n_d}$ an exogenous plant disturbance and $p(t) \in \mathbb{R}^l$ a time-varying parameter. The system matrices $A(p)$ and $B(p)$ belong to the polytopic matrix family

$$\Sigma(\mathcal{P}) := \{(A(p), B(p)) = \sum_{i=1}^{l} p_i (A_i, B_i),\ p \in \mathcal{P}\} \qquad (2)$$

where the couple (A_i, B_i) denotes the vertices of the polytope $\Sigma(\mathcal{P})$, viz. $(A_i, B_i) \in \text{vert}\{\Sigma(\mathcal{P})\}$, $\forall i \in \underline{l} := \{1, 2, \ldots, l\}$. The disturbance $d(t)$ belongs to the following set

$$d(t) \in \mathcal{D} \subset \mathbb{R}^{n_d}, \qquad \forall t \in \mathbb{Z}_+ \qquad (3)$$

with $\mathcal{D}$ compact with $0_n \in \mathcal{D}$. The vector parameter $p = [p_1, \cdots, p_l]'$ is assumed to belong to the unit simplex

$$\mathcal{P} := \{p \in \mathbb{R}^l : \sum_{i=1}^{l} p_i = 1,\ p_i \geq 0\} \qquad (4)$$

and we will suppose the following property to hold true

$$\textbf{LPV} - p(t) \text{ is measurable at each time instant} \qquad (5)$$

Moreover, the following input saturation constraints are prescribed

$$u(t) \in \mathcal{U} \qquad \forall t \geq 0 \qquad (6)$$

where $\mathcal{U}$ is a convex compact set such that $0_u \in \mathcal{U}$. Then, the problem we want to solve can be formulated as:

MPC Problem - Given the linear time varying system (1)-(4), compute at each time instant t and on the basis of the current state $x(t)$ and the actual parameter measure $p(t)$, a control strategy $u(t) = g(x(t), p(t))$ compatible with (6) such that a convenient one-step running cost $J(x(t), u(\cdot))$ is minimized. □

III. Open-loop state trajectories

The LPV hypothesis can be exploited in the computation of the feedback control law $u(t) = g(x(t))$ because the parameter strongly affects the plant evolution and it is a relevant information for a possible less conservative control action. For such a reason, linearly scheduled control laws will be used having the following expression

$$u(p(t)) = \sum_{j=1}^{l} p_j(t)\, u_j,\ \ u_j \in \mathbb{R}^m \qquad (7)$$

where the state dependence of u_j will be put in evidence in section IV. It is worth to note that if we substitute (7) into (1) the one-step ahead state evolution depends on the square of the parameter vector p. Such a nonlinearity would result in prediction set machineries and stabilizability conditions which are not easy to be numerically managed. For such a reason it is convenient to proceed with a proper convexification of the state evolutions set. In [18] and [7] the "half-sum" approach has been used which gives rise to less conservative stabilizability conditions than the ones standard quadratic ones. The key idea was to rewrite the one-step ahead state evolution as follows

$$\begin{aligned} x(t+1) = &\sum_{i=1}^{l} p_i^2(t)(A_i x(t) + B_i\, u_i) + \\ &\sum_{\substack{i=1,\ldots,l \\ j>i}} 2p_i(t)p_j(t) \frac{(A_i + A_j)x(t) + (B_i u_j + B_j u_i)}{2} x(t) \end{aligned} \qquad (8)$$

with the vector $[p_1^2, \ldots p_l^2, 2p_1p_2, \ldots 2p_{l-1}p_l]$ belonging to the $l(l+1)/2$ unit simplex

$$\sum_{i=1}^{l} p_i^2(t) + \sum_{i=1}^{l} \sum_{j>i} 2p_i(t)p_j(t) = 1. \qquad (9)$$

The above convexification can eventually be improved by a judicious sharpening of the unitary simplex (9). To this end, let us introduce the following auxiliary result:

Lemma 1 - Let $p(t) \in \mathcal{P}$, where $\mathcal{P}$ denotes the unit simplex (4). Then, the following property holds true

$$2\, p_i(t)\, p_j(t) \leq \frac{1}{2},\ \ i = 1, \ldots, l,\ j = i+1, \ldots, l. \qquad (10)$$

Proof - Let us consider the maximum allowable value of a single term $p_i\, p_j$ which is obtained when $p_k = 0$, $\forall k \neq i, j$. Under this condition and because $p \in \mathcal{P}$, one has that $p_i + p_j = 1$. Therefore, it follows that $p_i\, p_j = (1 - p_j)p_j = p_j - p_j^2$. As a consequence the maximum value of the above equation is attained when $p_j = \frac{1}{2}$ and (10) follows. Note that the result (10) can be also proved by resorting to the Pólya's Theorem [15]. □

The rationale of **Lemma 1** is that the parameter $p(t)$ can be embedded into a new convex region defined by the intersection of the unit simplex (9) and the family of half-spaces (10).

Next figure 1 gives a graphical explanation of **Lemma 1** arguments when $l = 2$. Points (A, B, C) represent the vertices of the unit simplex (9), while the couple (D_1, D_2) describes the points of intersection between the unit simplex (9) and the hyperplane $2\,p_1 p_2 = 1/2$. The dashed line in Figure 1 depicts the projection of the parameter vector $p(t)$ lying in the $(p_1^2, p_2^2, 2p_1p_2)$ space.

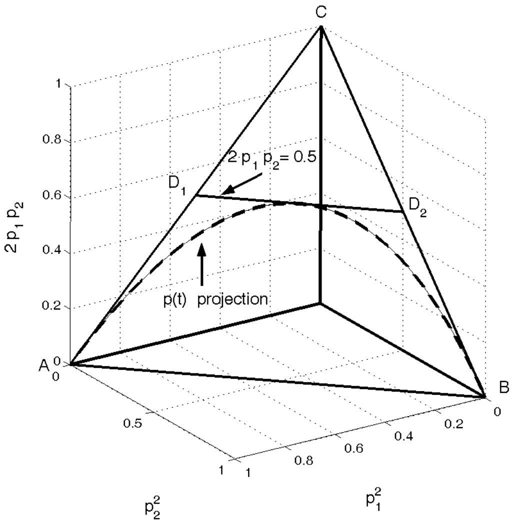


Fig. 1. Projection of $p(t)$ on the $(p_1^2, p_2^2, 2p_1p_2)$ space (Dashed line graph)

By exploiting **Lemma 1**, the closed-loop system described by (8)-(9) can be suitably rewritten. In fact, condition (10) allows one to intersect the hyperplanes

$$2\,p_i(t)\,p_j(t) = \frac{1}{2},\;\; i = 1,\ldots,l,\; j = i+1,\ldots,l$$

with the unit simplex (9) and one can rewrite (8) as follows

$$\begin{aligned}
x(t+1) =& \sum_{i=1}^{l} \bar{p}_i(t)\,[A_i x(t) + B_i u_i] + \\
&\sum_{\substack{i=1,\\k=1,\\s=k+1}}^{l} \bar{p}_{iiks}(t)\left[\frac{(2A_i+A_k+A_s)x(t)+2B_iu_i+B_ku_s+B_su_k}{4}\right] + \\
&\sum_{\substack{i=1,\\j=i+1,\\s=j+1}}^{l} \bar{p}_{ijis}(t)\left[\frac{(2A_i+A_j+A_s)x(t)+B_iu_j+(B_j+B_s)u_i+B_iu_s}{4}\right] + \\
&\sum_{\substack{i=1,\\j=i+1,\\k=i+1,\\s=k+1}}^{l} \bar{p}_{ijks}(t)\left[\frac{(A_i+A_j+A_k+A_s)x(t)+B_iu_j+B_ju_i+B_ku_s+B_su_k}{4}\right]
\end{aligned} \tag{11}$$

with

$$\sum_{i=1}^{l}\bar{p}_i(t) + \sum_{\substack{i=1,\\k=1,\\s=k+1}}^{l} \bar{p}_{iiks}(t) + \sum_{\substack{i=1,\\j=i+1,\\s=j+1}}^{l} \bar{p}_{ijis}(t) + \sum_{\substack{i=1,\\j=i+1,\\k=i+1,\\s=k+1}}^{l} \bar{p}_{ijks}(t) = 1 \tag{12}$$

where $\bar{p}_i(t) \geq 0$ and $\bar{p}_{ijks}(t) \geq 0$ are new suitable combinations of the parameter vector components $p_i(t), i = 1,\ldots,l$ defining the unit simplex (9).

Remark 1 - The proposed convexification is obviously less conservative w.r.t. (8). The price to be paid is an increased computational complexity: the number of vertices becomes $\frac{l^4+2l^3-5l^2+10l}{8}$ instead of $l(l+1)/2$. □

Remark 2 - It is worth to note that both the LPV convexifications (8) and (11) are equivalent to robust polytopic systems with state $x(t) \in \mathbb{R}^n$ and input vector $[u_1^T,\ldots,u_l^T]^T \in \mathbb{R}^{lm}$. □

IV. An Ellipsoidal Off-line MPC Scheme

A convenient way of addressing the problem described in Section II in a MPC context is to resort to a dual-mode scheme. First, a stabilizing LPV control law for (1) within a suitable neighborhood of the operating point is derived, then the working region of the algorithm is possibly extended by off-line computing sets of states that can be steered into the terminal set in a finite number of steps for each parameter occurrence and despite of disturbances. To develop the proposed strategy, some notation is necessary. We denote by $\mathcal{A} \sim \mathcal{B} := \{a : a + b \in \mathcal{A}\ \forall\ b \in \mathcal{B}\}$ the *subtraction* between sets [14]. The set $\mathcal{T}$ is said to be *positively robustly invariant* w.r.t. the system (1) subject to a state feedback control law

$$u(x(t), p(t)) = F(p)x(t) = \sum_{j=1}^{l} p_j(t)\,F_j\,x(t) \tag{13}$$

if, for any $x \in \mathcal{T}$, any $d \in \mathcal{D}$ and $p \in \mathcal{P}$ then $x(t+1) \subset \mathcal{T}$. Given a robustly controlled-invariant region $\mathcal{T}$ it is possible, in principle, to compute the sets of states i steps controllable to $\mathcal{T}$, regardless of disturbances and for each parameter occurrence acting on the system, via the following recursion:

$$\mathcal{T}_0 \;:=\; \mathcal{T} \tag{14}$$

$$\begin{aligned}\mathcal{T}_i \;:=\; &\{x \in \mathbb{R}^n : \exists\, u(p) \in \mathcal{U} : \forall\, d \in \mathcal{D},\ \forall p \in \mathcal{P}, \\ &(A(p)x + B(p)u(p) + G_d d) \in \mathcal{T}_{i-1}\}\end{aligned} \tag{15}$$

where $\mathcal{T}_i$ is the set of states that can be steered in a single control move into $\mathcal{T}_{i-1}$, irrespective of disturbances and using the parameter dependent control $u(p)$ introduced in (7). As a consequence, by induction, we have that $\mathcal{T}_i$ is the set of states that can be steered into $\mathcal{T}$ within at most a number of i control moves. Many properties of $\mathcal{T}_i$ have been investigated in [5]. As discussed in [1], the shapes of the $\mathcal{T}_i$ grow in complexity as the index i grows and may become computationally intractable after a small number of iterations. For this reason we define a variant of the recursion (15) which exploits ellipsoidal inner approximations $\{\mathcal{E}_i\}_{i=0}^{\bar{i}}$, thus allowing a constant number of parameters at each iteration for their characterization. In particular, each member $\mathcal{E}_i$ represents a compact set of states that can be steered using a parameterized control move

$u(p)$, for each occurrence of p, irrespective of disturbances and without constraints violation, into another member $\mathcal{E}_{i-1}$. After a finite number of steps $\bar{i}$ the regulated state trajectories is then confined into a suitable robust terminal set.

Proposition 1 - Let $\mathcal{E} \subset \mathbb{R}^n$ be a nonempty robustly invariant ellipsoidal region under the state-feedback control law (13):

$$(A_i + B_i F(p)))\mathcal{E} + G_d\mathcal{D} \subset \mathcal{E} \tag{16}$$

for all $i = 1, 2, \ldots l$. Then, the sequence of ellipsoidal sets

$$\begin{array}{rcl} \mathcal{E}_0 & := & \mathcal{E} \\ \mathcal{E}_i & := & \text{In}[\{x : \exists\, u(p) \in \mathcal{U} : \forall\, d \in \mathcal{D}, \forall\, p \in \mathcal{P}, \\ & & A(p)x + B(p)u(p) + G_d d \in \mathcal{E}_{i-1}\}] \end{array} \tag{17}$$

if non-empty, satisfies $\mathcal{E}_i \subset \mathcal{T}_i$. In (17), with the notation In$[\cdot]$ we mean an inner ellipsoidal approximation.

Proof - See [1]. □

Remark 3 - It is worth to observe that the maximum number $\bar{i}$ of ellipsoidal sets to be computed depends on the problem at hand. If $x(0)$ is given, it is enough that $x(0) \in \bigcup_{i=0}^{\bar{i}} \mathcal{E}_i$. More generally, $\bigcup_{i=0}^{\infty} \mathcal{E}_i$ represents the attraction basin of our algorithm, that is the set of all initial states for which we guarantee the existence of a solution. □

The ellipsoids $\mathcal{E}_i$ can be numerically derived, for instance, by means of LMIs as been proposed in [1] for the robust case. The same technicalities can be adapted by exploiting the convexification (11)-(12) to describe the one-step ahead state prediction $\hat{x}(t+1|t)$.

The above developments allow one to easily derive a Receding Horizon Control strategy. On-line, at each time t, the smallest index i such that $x(t) \in \mathcal{E}_i$ is first computed. Then, such a sequence of sets is used to enforce, at each step, $\hat{x}(t+1|t) \in \mathcal{E}_{i-1}$ in order to ensure contraction and viability to the closed-loop trajectories. Then, the MPC algorithm is as follows:

Algorithm LPV-NS -

Off-line -

0.1 Compute the robustly invariant ellipsoidal region $\mathcal{E}_0$ and the parameter scheduling state feedback law $F(p(t))$ (13) complying with (16);

0.2 Generate a sequence of N one-step controllable sets $\mathcal{E}_i$ (17) by using the procedure described in detail in [1];

0.3 Store $F(p(t))$ and $\mathcal{E}_i,\ i = 0, \ldots, N$.

On-line -

1.1 Let $i(t) := \min\{i : x(t) \in \mathcal{E}_i\}$

1.2 If $i(t) = 0$ then $u(t) = F(p(t))x(t)$

1.3 Else,

$$\begin{array}{l} u(t) = \arg\min J_{i(t)}(x(t), u(t)) \\ \ \text{subject to} \\ \Phi(p(t))x(t) + G(p(t))u(t) \in \text{In}\big[\mathcal{E}_{i(t)-1} \sim G_d\mathcal{D}\big]\,, \\ u(t) \in \mathcal{U}, \end{array}$$

1.4 apply $u(t)$; $t := t + 1$; goto 1.1;

Remark 4 - The main merit of an LPV framework is that the one-step ahead state prediction is exactly known at each time instant whereas in the robust case the prediction belongs to a polyhedron. Such a feature is particularly well exploited within the proposed algorithm during the on-line phase, providing both a reduction in the computational cost and a control performance improvement. □

Notice also that the ellipsoidal sets In $\big[\mathcal{E}_{i(t)-1} \sim G_d\mathcal{D}\big]$ in step 1.3. of the proposed algorithm can be computed off-line and properly stored for the on-line use. Finally observe that the cost $J_{i(t)}(x(t), u)$ may depend on $i(t)$ and be defined on an infinite horizon if e.g. implicit dual-mode MPC schemes are of interest. Otherwise, typical choices include

$$J_{i(t)}(x(t), u) = \max_j \|\Phi_j x(t) + G_j u\|^2_{Q_{11}^{-1}(i(t)-1)}$$

where $\mathcal{E}_i = \{x :\ x' Q_{11}^{-1}(i)x \leq 1\}$, or

$$J_{i(t)}(x(t), u) = \|u\|^2_{\Psi},\ \Psi = \Psi^T > 0$$

if one is interested to approximate one-step minimum-time or, respectively, one-step minimum-energy algorithms. Due to the fact the optimization problem in step 1.3 is always feasible, the following stability result holds true.

Proposition 2 - Let the sequence of ellipsoids $\mathcal{E}_i$ be non-empty and $x(0) \in \bigcup_i \mathcal{E}_i$. Then, the algorithm **LPV-NS** always satisfies the constraints and ensures robust stability. In particular there exists a finite time $\bar{t}$ such that $x(t) \in \mathcal{E}_0$ for all $t \geq \bar{t}$.

Proof - The proof follows from Propositions 1. Convergence to $\mathcal{E}_0$ in a finite time follows from the choice of a finite number $\bar{i}$ of ellipsoids $\mathcal{E}_i$. □

V. Numerical example

The aim of this section is to present results on the effectiveness of the proposed MPC strategy and to give a measure of the improvement deriving from the use of the hypothesis (5) and from the state evolutions (11)-(12). All computations have been carried out on a PC Intel^{TM} Centrino with the Matlab© LMI Toolbox.

We make comparisons of the **LPV-NS** scheme with the robust counterpart algorithm [1] (**Robust**) and with the same LPV algorithm when the state evolutions (8)-(9) proposed in [18] are put in place of (11)-(12), hereafter named **LPV-SS**. To this end, consider the multi-model linear time-varying system

$$x(t+1) = \sum_{i=1}^{2} p_i(t)A_i\, x(t) + \sum_{i=1}^{2} p_i(t)B_i\, u(t) \tag{18}$$

with

$$A_1 = \begin{bmatrix} 2 & -0.1 \\ 0.5 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 0.1 \\ 2.5 & 1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ -0.3 \end{bmatrix}, B_2 = \begin{bmatrix} 0.7 \\ 0.1 \end{bmatrix}, \tag{19}$$

and input saturation constraint $|u(t)| \leq 1,\ \forall t \geq 0$. The parameter vector $p(t)$ is assumed to be measurable at each

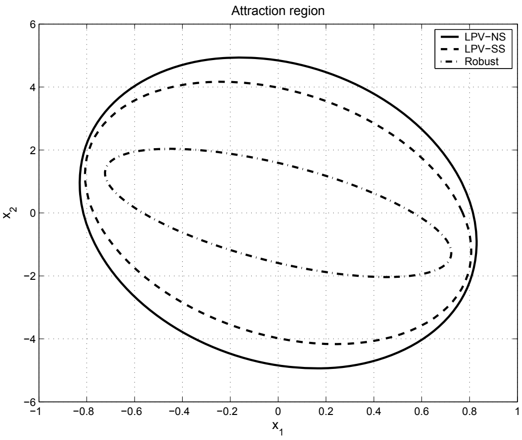


Fig. 2. State Attraction Region with input bound constraints - **LPV-NS** (Continuous line), **LPV-SS** (Dashed line), Robust (Dash-Dotted line)

time instant t. In particular the time-varying parameter realization $p(t) = [sin(t)\ 1 - sin(t)]$ and the initial state $x_0 = [-0.7 \quad 1.5]^T$ (admissible for all the strategies, see Fig. 2) have been supposed. Two control strategies have been taken here into consideration: approximate minimum-time and approximate minimum-energy. Then, a family of 50 ellipsoids has been generated for each algorithm.

The basins of attraction for the three algorithms are depicted in Figure 2. As it clearly results, an enlarged region of feasible initial states results for the proposed algorithm (continuous line).

In Table I the CPU times for the off-line phases have been reported. Essentially all the three algorithms have similar computation times.

TABLE I
NUMERICAL BURDENS: OVERALL CPU TIME (SECONDS)

Algorithm	Off-line: Overall CPU Time	On-line: Average CPU time (seconds per step)
LPV-NS	17.297	0.0870
LPV-SS	16.282	0.0849
Robust	14.624	0.1854

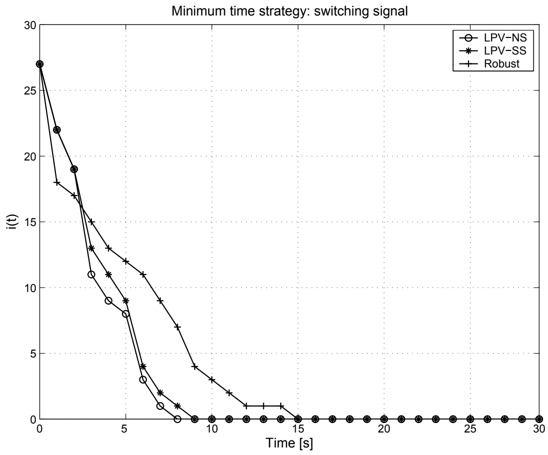
The average on-line numerical complexity for each algorithm is reported in Table I. It is possible to observe that one order of magnitude separates the LPV methods from the robust ellipsoidal scheme. This is due to the fact that the robust case implies that the minimization (Step 1.3 of the algorithm) is performed along the corners characterizing the one-step ahead state prediction while it reduces to a single point in the LPV framework.

Next comparisons are in terms of (approximate) minimum-time and minimum-energy control performance.

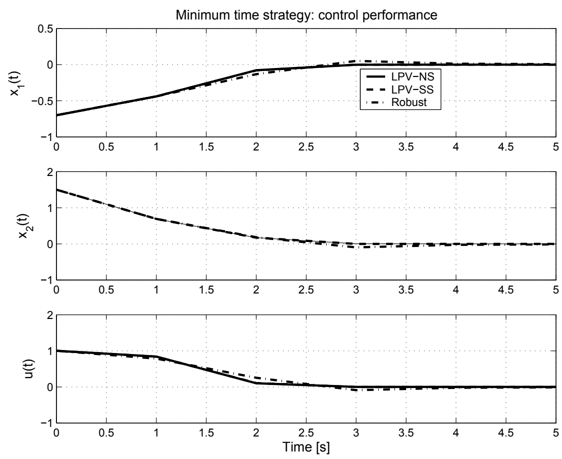
A. Minimum time strategy

All the relevant results are depicted in Fig. 3. Fig. 3(a) depicts the switching signal $i(t)$ for all the contrasted algorithms. This signal is important to show the level of contraction provided by the control algorithm during the system evolution and it represents at each time instant the smaller ellipsoid $\mathcal{E}_i$ of the pre-computed family containing the state $x(t)$. The state and command behaviours corresponding to this realization are depicted in Fig. 3(b).

Under the minimum-time criterion, the two LPV strategies perform almost identically, while a slight worst behaviour is observed in the case of the robust counterpart. In fact shorter settling times can be observed in Fig. 3(b). This is confirmed by the switching signal $i(t)$ trend Fig. 3(a).



(a)

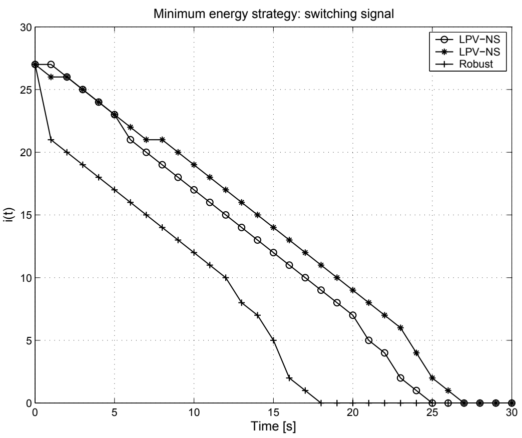


(b)

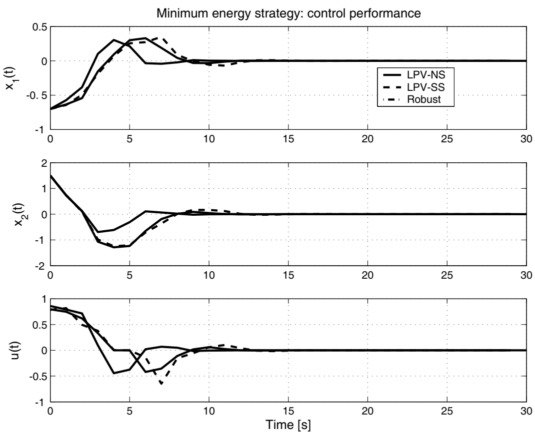
Fig. 3. (Left) Switching signals for the proposed strategy: one-step minimum-time. (Right) Regulated state evolutions and commands: Minimum-time strategies

B. Minimum energy strategy

Under the minimum-energy criterion, all the simulations are reported in Figs. 4-5. In this case, unlike the minimum-time criterion, the proposed **LPV** algorithm performs significantly better than the other two algorithms (see Fig. 4(b)). The improvement of the proposed strategy becomes more evident when cumulative input energy plots $\sum_{i=0}^{t} u(i)^2$ are considered, see Fig. 5. In fact, the steady-state value reached by the **LPV-NS** algorithm is evidently the lowest w.r.t. the others.



(a)



(b)

Fig. 4. (Left) Switching signals for the proposed strategy: one-step minimum-energy. (Right) Regulated state evolutions and commands: Minimum-energy strategies

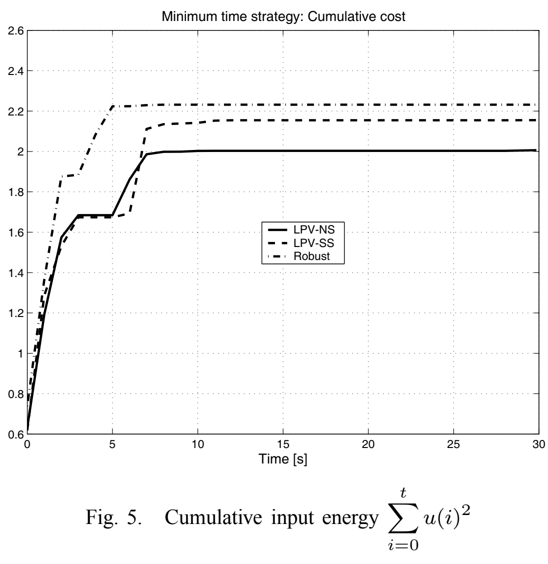


Fig. 5. Cumulative input energy $\sum_{i=0}^{t} u(i)^2$

VI. Conclusion

A novel receding horizon control strategy for multi-model LPV systems has been described under persistent disturbances and subject to input constraints. The control strategy is an extension to the LPV case of a low-demanding robust receding horizon control scheme recently proposed in literature. The algorithm here proposed takes advantage both of the availability of the parameter vector at each time instant and a tighter refinement of the state trajectories tube w.r.t. traditional approaches.

Comparisons with an alternative LPV version of the same off-line solution and robust counterparts have been taken into consideration in the final numerical section.

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