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New stabilizability conditions for discrete-time Linear Parameter Varying systems

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Abstract— **New stabilizability conditions for input-saturated multi-model Linear Parameter Varying (LPV) discrete-time systems are proposed. They are obtained by means of a "refinement" of the usual quadratic stabilizability conditions. It is proven that such conditions can be made significantly more tight by a better characterization of the politopic set which embeds the true LPV parameter realization. An application to a computationally low demanding "off-line" Receding Horizon Control (RHC) scheme is provided. A numerical example is also presented in order to show that the regulation strategy based on the refined conditions significantly improves the performance of the control strategy and its basin of attraction.**

I. INTRODUCTION

One of the fundamental problems arising in constrained control theory is the derivation of an efficient feedback control strategy capable to jointly stabilize (i.e. regulation at the desired set point) the possibly uncertain system and guarantee a certain level of performance. Important results are scattered in the vast literature related to this area which addresses interesting practical topics for control synthesis. These statements are supported by a number of articles, tutorials and books as [7], [6], [5] and references therein, to mention a few.

Multi-Model LPV systems are by no means one of new emerging paradigms used to rigorously capture the behaviour of a certain class of nonlinear systems and/or give soundness to gain-scheduling control approaches (see [1], [2]). Strictly speaking, such a LPV system can be viewed as a linear system whose parameters in the state space representation can be denoted by a polytopic or an affine combination of a set of fixed vertices. Some of the parameters are assumed to be time-varying and measurable at each time instant.

For this class of systems the stabilizability issue has been addressed in literature by resorting to classical quadratic stability concepts: a single couple of feedback gain and Lyapunov matrix is selected so as to simultaneously stabilize all plant vertices [11]. It is straightforward to note that such an approach does not take explicitly into consideration the property regarding the measurability of the system parameters and the resulting control law is inherently conservative.

Another way to attack the problem is to take directly into account the system parameter in the design phase. In this case, a time-varying control law is achieved, suitably tuned at each time on the current measure of the parameter. As a consequence, the proposed regulation strategy has a scheduling feature and a more satisfactory regulation performance (see [10],[9]).

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The drawbacks of the above gain-scheduling regulation framework are well known. In particular the number of LMI conditions grows with the square of the plant vertices whereas only linearly in the standard quadratic framework. Moreover, such conditions define an extremely rough polytopic outer approximation of the true closed-loop state evolutions. Some improvements on the tightness of the above approximation have been achieved by considering the combination of plant/controller vertices in "half-sums" (see [20]).

Here we go further on and show that a substantial reduction of conservativeness can be achieved by a simple argument underlying the structure of the scheduling parameters which apparently have not been yet considered in the literature.

As an example of application, we will consider a revisitation of the recent Receding Horizon Control (RHC) strategy of [19] which will be extended to the LPV framework, equipped with the stabilizability conditions of [20] and contrasted with the same control law modified with the new stabilizability conditions proposed here. A numerical example will show the improvements in terms of basin of attraction, regulated response and guaranteed cost index.

The paper is organized as follows: the problem is stated in Section II where previous relevant results are summarized. In Section III, the new stabilizabilty conditions are stated and the main result proved. In Section IV the conditions are applied to a RHC problem and its feasibility/stability properties proved. Finally a numerical example is reported in Section V and some conclusions end the paper.

II. PROBLEM FORMULATION AND PREVIOUS RESULTS

Consider the following uncertain discrete-time linear system

$$\left\{ \begin{array}{lcl} x(t+1) & = & A(p(t))\,x(t) + B(p(t))\,u(t) \\ y(t) & = & Cx(t) \end{array} \right. \tag{1}$$

with $x(t) \in \mathbf{R}^n$ denoting the state, $u(t) \in \mathbf{R}^m$ the plant input and $y(t) \in \mathbf{R}^p$ the output. The plant matrices have the following structure:

$$A(p) = \sum_{j=1}^{l} p_j A_j, \quad B(p) = \sum_{j=1}^{l} p_j B_j, \tag{2}$$

with $p = [p_1, p_2, \ldots, p_l]^T \in \mathcal{P}$ in the unit simplex

$$\mathcal{P} := \left\{ \sum_{j=1}^{l} p_j = 1,\ 0 \le p_j \le 1 \right\}. \quad (3)$$

Here, we restrict our attention to LPV systems:

$$\textbf{LPV} - p(t) \text{ is measurable at each time instant.} \quad (4)$$

We will now state the classical stabilizability problem [7] by supposing the **LPV** property to hold true.

Problem ST - *Given the LPV discrete-time system (1)-(3), determine a linear state feedback control law $u(t) = F\,x(t)$, such that the closed-loop system matrix $A(p(t)) + B(p(t))\,F$ is asymptotically stable for any $p(t) \in \mathcal{P}$.*

We begin our discussion by recalling that (robust) quadratic stabilization of the polytopic system (1)-(3) is equivalent to the stabilization of its vertices $(A_i,\ B_i)\, i = 1, \ldots, l$.

Lemma 1 (Geromel et al., 1991 [11]) - The uncertain multi-model linear system (1)-(3) is robustly quadratically stabilizable by a linear state feedback control law $u(t) = F\,x(t)$ if there exists a couple of matrices $(P,\ F)$, $P = P^T > 0$ such that

$$(A_i + B_i\,F)^T\,P\,(A_i + B_i\,F) - P \le 0,\ \ i = 1, \ldots, l. \quad (5)$$

□

Note that conditions (5) can be converted into a set of l LMIs (see [7]) and solved via standard semidefinite programming machinery.

If the parameter vector $p(t)$ is measurable at each time instant t then a parameter-dependent state feedback law

$$F(p(t)) = \sum_{j=1}^{l} p_j(t)\,F_j \quad (6)$$

with $F_i,\ i = 1, \ldots, l$ denoting a family of controllers, can be used to solve **Problem ST** and improve the control performance. Under (6), the stabilizability conditions (5) translate into

$$(A_i + B_i\,F_j)^T\,P\,(A_i + B_i\,F_j) - P \le 0,\ \ i,j = 1, \ldots, l \quad (7)$$

All control paradigms based on the previous set of inequalities allow one to reduce the intrinsic conservativeness of quadratic stabilizability at a price of an increased computational complexity; the cardinality of the LMI set becomes in fact quadratic (l^2) in the plant vertices.

In [20], [8], the stabilizability conditions have been further improved via a "half-sum" trick. The key idea was to rewrite the closed-loop uncertain system as follows

$$\begin{aligned} x(t+1) =& \sum_{i=1}^{l} p_i^2(t)(A_i + B_i\,F_i)x(t) \\ &+2 \sum_{\substack{i=1,\ldots,l \\ j>i}} p_i(t)p_j(t)\frac{(A_i + B_i\,F_j) + (A_j + B_j\,F_i)}{2}x(t) \end{aligned} \quad (8)$$

with

$$\sum_{i=1}^{l} p_i^2(t) + 2\sum_{i=1}^{l}\sum_{j>i} p_i(t)p_j(t) = 1, \quad (9)$$

then the following result holds true.

Lemma 2 (Casavola et al, 2003 [8]) - The multi-model LPV system (1)-(3) is quadratically stabilizable by a parameter scheduling state feedback law (6) if there exists a $l+1$-tuple $(P,\ \{F_j\}_{j=1}^{l})$, $P = P^T > 0$ such that

$$\begin{aligned}&\left(\tfrac{(A_i+B_i\,F_j)+(A_j+B_j\,F_i)}{2}\right)^T P \left(\tfrac{(A_i+B_i\,F_j)+(A_j+B_j\,F_i)}{2}\right) \\ &-P \le 0,\ \ i = 1, \ldots, l,\ j = i, \ldots, l.\end{aligned} \quad (10)$$

□

Remark 1 - It is worth to note that the improvement underlying conditions (10) is twofold. First, the conservativeness of the control design is reduced and then the number of LMI conditions to be checked is reduced to $\frac{l(l+1)}{2}$ vs l^2 of standard quadratic stabilizability (7). □

Next section is devoted to derive our main result which allows a further improvement.

III. Main result

The key idea underlying Lemma 2 is that a better understanding of the parameter vector time behavior $p(t)$ can be used to reduce the size of the convex hull embedding the closed-loop state trajectories. In fact, from (8)-(9) it follows that the the scheduling parameter $p(t)$ is embedded into a smaller $\frac{l(l+1)}{2}$−dimensional polytope contained in the unit simplex.

Here, such a convex set will be refined further on. Consider first the following preliminary result.

Lemma 3 - Let $p(t) \in \mathcal{P}$, with $\mathcal{P}$ denoting the unit simplex (3). Then, the following conditions hold true

$$2\,p_i(t)\,p_j(t) \le \frac{1}{2},\ \ i = 1, \ldots, l,\ j = i+1, \ldots, l. \quad (11)$$

Proof - Note that the statement of Lemma 3 represents an alternative formulation of Pólya's Theorem [16],[17] and here a simpler proof will be provided. To this end, consider the maximum allowable value of a single term $p_i\,p_j$ which is obtained when $p_k = 0,\ \forall k \ne i, j$. Under this condition and because $p \in \mathcal{P}$, one has that $p_i + p_j = 1$. Therefore, it follows that $p_i\,p_j = (1-p_j)p_j = p_j - p_j^2$. As a consequence the maximum value of the above equation is obtained when $p_j = \frac{1}{2}$ and (11) follows. □

The main result of the previous lemma is that the parameter $p(t)$ can be embedded into a new convex region defined by the intersection of the unit simplex (9) and the family of half-spaces

$$2\,p_i(t)\,p_j(t) \le \frac{1}{2},\ \ i = 1, \ldots, l,\ j = i+1, \ldots, l. \quad (12)$$

Next figure 1 reports a schematic picture of the geometry underlying **Lemma 3** when $l = 2$. The points (A, B, C) represent the vertices of the unit simplex (9), while the couple

(D_1, D_2) describe the points of intersection between the unit simplex (9) and the hyperplane

$$2\, p_1 p_2 = \frac{1}{2}.$$

Note that the dashed line depicts the projection of the parameter vector $p(t)$ lying in the $(p_1^2, p_2^2, 2p_1p_2)$ space.

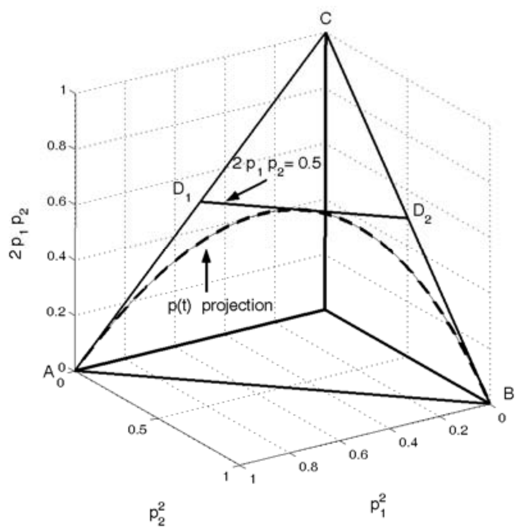

Fig. 1. Projection of $p(t)$ on the $(p_1^2, p_2^2, 2p_1p_2)$ space (Dashed line graph)

□

In view of **Lemma 3**, the closed-loop system described by (8)-(9) can be suitably rewritten. In fact, condition (11) allows one to intersect the hyperplanes

$$2\, p_i(t)\, p_j(t) = \frac{1}{2},\;\; i = 1, \ldots, l,\; j = i+1, \ldots, l$$

with the vertices of the unit simplex (9). By defining the following matrices

$$\Phi_{i,j} := \frac{1}{2}\left((A_i + B_i\, F_j) + (A_j + B_j\, F_i)\right) \tag{13}$$

one can rewrite (8) as follows

$$x(t+1) = \left(\sum_{i=1}^{l} \bar{p}_i(t)\Phi_{i,i} + \sum_{\substack{i=1,\\ j=i+1}}^{l} \sum_{\substack{s=1,\\ k=s,\\ (i,j)\neq(k,s)}}^{l} \bar{p}_{ijks}(t) \frac{\Phi_{i,j} + \Phi_{k,s}}{2} \right) x(t) \tag{14}$$

with

$$\sum_{i=1}^{l} \bar{p}_i(t) + \sum_{\substack{i=1,\\ j=i+1}}^{l} \sum_{\substack{s=1,\\ k=s,\\ (i,j)\neq(k,s)}}^{l} \bar{p}_{ijks}(t) = 1 \tag{15}$$

where $\bar{p}_i(t)$ and $\bar{p}_{ijks}(t)$ are suitable combinations of $(p_i(t)\, p_j(t)\, p_k(t)\, p_s(t))$. Then, our main result on the stabilizability of the LPV system (1)-(3) can be stated.

Proposition 1 - The LPV polytopic system (1)-(3) is quadratically stabilizable by a parameter scheduling state feedback law (6) if there exists a $l+1$-tuple $(P, \{F_j\}_{j=1}^{l}), P = P^T > 0$ such that

$$(\Phi_{i,i})^T\; P\, (\Phi_{i,i}) - P \leq 0, \qquad i = 1, \ldots, l, \tag{16}$$

$$\begin{array}{c} \left(\frac{1}{2}\Phi_{i,j} + \frac{1}{2}\Phi_{k,s}\right)^T\; P \left(\frac{1}{2}\Phi_{i,j} + \frac{1}{2}\Phi_{k,s}\right) - P \leq 0, \\ i = 1, \ldots, l,\;\; j = i+1, \ldots, l, \\ k = 1, \ldots, l,\;\; s = k, \ldots, l,\;\; (i,j) \neq (k,s) \end{array} \tag{17}$$

Proof - The result follows by taking into account the standard Lyapunov inequality

$$(A + BF)^T P (A + BF) - P \leq 0$$

and using convexity arguments on the closed-loop trajectories (14). □

Moreover, it is straightforward to recast conditions (16)-(17) as LMIs.

Corollary 1 - The LPV polytopic system (1)-(3) is quadratically stabilizable by a parameter scheduling state feedback law (6) if there exists a $l+1$-tuple $(P, \{F_j\}_{j=1}^{l}), P = P^T > 0$ such that

$$\left[\begin{array}{cc} Q & * \\ \Phi_{i,i}^Y & Q \end{array} \right] \geq 0, \tag{18}$$

$$i = 1, \ldots, l.$$

$$\left[\begin{array}{cc} Q & * \\ \frac{1}{2}\Phi_{i,j}^Y + \frac{1}{2}\Phi_{k,s}^Y & Q \end{array} \right] \geq 0, \tag{19}$$

$$\begin{array}{c} i = 1, \ldots, l,\;\; j = i+1, \ldots, l, \\ k = 1, \ldots, l,\;\; s = k, \ldots, l,\;\; (i,j) \neq (k,s) \end{array}$$

where

$$\Phi_{i,j}^Y = \frac{(A_i Q + B_i\, Y_j) + (A_j Q + B_j\, Y_i)}{2} \tag{20}$$

and $Q = P^{-1},\; F_i = Y_i\, Q^{-1},\; i = 1, \ldots, l.$ □

Remark 2 - The idea here developed can be simply extended to deal with the recent dilated conditions proposed in [10], [9], where the use of parameter dependent Lyapunov functions (PDLF) ensures less conservative stability inequalities. □

Remark 3 - It is worth pointing out that any $l+1$ tuple that satisfies the inequalities (10) is an admissible solution for (16)-(17). Such a property guarantees that the proposed conditions are less conservative than all previous cited ones for quadratic stabilizability. The price to be paid is an increased computational complexity: in fact the number of LMI constraints to be checked becomes $\frac{l^4 - 3l^2 + 6l}{4}$ instead of $l(l+1)/2$ as in (10). □

Remark 4 - In order to appreciate the improvement of the proposed conditions, we shall consider the problem to find the largest value of a positive scalar $\bar{\alpha}$ such that the following affine LPV system

$$x(t+1) \;\; = \;\; (A_0 + \alpha A_1)x(t) + (B_0 + \alpha B_1)u(t) \tag{21}$$

where

$$\begin{array}{c} A_0 = \left[\begin{array}{cc} 1.5 & 0 \\ 1.5 & 1 \end{array} \right],\; B_0 = \left[\begin{array}{c} 0.85 \\ -0.1 \end{array} \right] \\ A_1 = \left[\begin{array}{cc} 0.5 & -0.1 \\ -1 & 0 \end{array} \right],\; B_1 = \left[\begin{array}{c} 0.15 \\ -0.2 \end{array} \right] \end{array} \tag{22}$$

admits a stabilizing scheduling feedback (6) for all $0 < \alpha < \bar{\alpha}$ by considering the quadratic stabilizability conditions (5), "half-sums" conditions (10) and the proposed new (16)-(17) respectively. The achieved bounds under the three cases are summarized in the following table. □

TABLE I
BOUNDS

Stabilizability conditions	$\bar{\alpha}$
(5)	3.2
(10)	4.45
(16)-(17)	5.6

Remark 5 - If $u(t) = 0$ the Lyapunov inequalities arising from the **Proposition 1** recover the classical form derived e.g. by Geromel et al. [11] for the quadratic stability. □

IV. An application: Receding Horizon Control

This section is devoted to show how the proposed stabilizability conditions are capable to reduce conservativeness in terms of overall control performance. To this end, a significant application is the stabilization problem of LPV systems subject to input-saturation constraints via a Receding Horizon Control (RHC) strategy.

The idea is to resort to a computationally low demanding RHC scheme where most of the computations are carried out off-line. Examples of such kind of strategies can be found in [3], [4], [12], [13], [15], [18] and [19]. Here the RHC algorithm proposed in [19] for uncertain polytopic systems will be extended to the LPV framework and used for comparisons.

RHC Problem - Given the system (1)-(3), determine at each time instant t, on the basis of the current state $x(t)$, a stabilizing pair $(P, F(t))$ which minimizes an upper bound to the cost index

$$J(x, u(\cdot)) := \max_{p(\cdot)\in\mathcal{P}^*} \sum_{t=0}^{\infty} \{\|x(t)\|^2_{R_x} + \|u(t)\|^2_{R_u}\}, \quad (23)$$

with $R_u = R_u^T > 0$, $R_x = R_x^T > 0$ suitable input and state weights, such that the following input constraints conditions are satisfied from t onward: $|u_j(t)| \leq u_{j,\max},\ t \geq 0, j = 1, \ldots, m.$ □

Because the control strategy has a time-varying linear state-feedback structure $F(t)$ as in (6), an upper-bound to the cost (23) is given by (see [14] for details)

$$J(x(t), u(\cdot)) \leq x(0)^T P\, x(t) \quad (24)$$

with $P = P^T > 0$ satisfying the (robust) quadratic stabilization conditions [11]. Then the ellipsoidal set $\mathcal{E}(P, \gamma) := \{x \in \mathbf{R}^n |\, x^T P x \leq \gamma,\ \gamma(t) \geq 0\}$ can be proved to be a robust positively invariant region for the polytopic system (1)-(3) under the scheduling based state-feedback control law $F(t)$.

A recent "off-line" solution to above **RHC Problem** has been achieved by Wan & Kothare [19]. That algorithm is here reported and adapted to the LPV framework using the stabilizability conditions of **Lemma 2**.

Algorithm-WK

Off-line -

0.1 Given an initial feasible state x_1, generate a sequence of minimizers γ^r, Q^r, Y^r, Z^r as follows

$$\min_{\gamma^r, Q^r, \{Y^r\}_{j=1}^l, Z^r} \gamma^r \quad (25)$$

subject to the constraints

$$\begin{bmatrix} Q^r & * & * & * \\ \frac{(A_i+A_j)Q^r+(B_iY_j^r+B_jY_i^r)}{2} & Q^r & 0 & 0 \\ R_x^{1/2}Q^r & 0 & \gamma^r I & 0 \\ R_u^{1/2}\frac{Y_i^r+Y_j^r}{2} & 0 & 0 & \gamma^r I \end{bmatrix} \geq 0,$$
$$\forall i = 1, \ldots, l,\ j = i, \ldots, l, \quad (26)$$

$$\begin{bmatrix} 1 & x' \\ x & Q^r \end{bmatrix} \geq 0 \quad (27)$$

$$Q^{r-1} > Q^r, \text{ (ignored at r=1)} \quad (28)$$

$$\begin{bmatrix} Z^r & Y_j^r \\ Y^{r\prime}{}_j & Q^r \end{bmatrix} \geq 0, \quad Z^r_{kk} \leq u^2_{k,\max},\ k \in \{1, 2, \ldots, m\}\ 1 \leq j \leq l, \quad (29)$$

with $F_j^r := Y_j^r(Q^r)^{-1},\ \forall j \in \{1, 2, \ldots, l\},\ P^r = \gamma^r\,(Q^r)^{-1}$.

0.2 Store $\gamma^r, Q^r, \{Y^r\}_{j=1}^l, Z^r$ in a look-up table;

0.3 If $r < N$, choose a state x_{r+1} satisfying

$$\|x_{r+1}\|_{(Q^r)^{-1}} < 1$$

Let $r = r + 1$, go to step 1.

On-line -

1.1 Given an initial feasible state $x(0)$ satisfying $\|x(0)\|_{(Q^1)^{-1}} \leq 1$, let the state be $x(t)$ at time t. Perform a bisection search over $(Q^r)^{-1}$ in the look-up table to find the largest index r such that

$$\|x(t)\|_{(Q^r)^{-1}} \leq 1$$

1.2 Feed the plant by the input

$$u(t) = \left(\sum_{j=1}^{l} p_j(t) F_j^r\right) x(t).$$

1.3 $t = t + 1$ and go to step 1.1.

In order to modify the above algorithm with the new stabilizability conditions consider the following result.

Lemma 4 - The LPV system (1)-(3) is quadratically stabilizable by the linear state-feedback control law

$$u(t) = \left(\sum_{j=1}^{l} p_j(t) F_j(t)\right) x(t)$$

and the upper bound (24) holds true if there exists a $l+1$-tuple $(Q, \{F\}_{i=1}^{l})$, $Q = Q^T > 0$ which satisfies the following linear matrix inequalities

$$Q > 0, \tag{30}$$

$$\begin{bmatrix} Q & * & * & * \\ \Phi_{i,i}^{Y} & Q & 0 & 0 \\ R_x^{1/2} Q & 0 & \gamma I & 0 \\ R_u^{1/2} Y_i & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \tag{31}$$

$$i = 1, \ldots, l,$$

$$\begin{bmatrix} Q & * & * & * \\ \frac{\Phi_{i,j}^{Y}+\Phi_{k,s}^{Y}}{2} & Q & 0 & 0 \\ R_x^{1/2} Q & 0 & \gamma I & 0 \\ R_u^{1/2}\,\frac{Y_i+Y_j+Y_k+Y_s}{4} & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \tag{32}$$

$$i = 1, \ldots, l, \quad j = i+1, \ldots, l,$$
$$k = 1, \ldots, l, \quad s = k, \ldots, l, \quad (i,j) \neq (k,s)$$

Proof - It follows the same standard arguments as in [14]. □

Remark 6 - Given the above result, a version of the above algorithm equipped with the new stabilizability conditions, hereafter denoted as **Algorithm-WK1**, can be obtained from **Algorithm-WK** by replacing the LMIs (26) with the two sets of LMIs (31) and (32). □

V. Numerical example

The aim of this example is to give a measure of the improvement deriving from the use of the new stabilizability LMI conditions (31)-(32) within a RHC framework. To this end, the RHC algorithm of [19] adapted to the LPV framework **Algorithm-WK** will be contrasted with the same RHC algorithm, **Algorithm-WK1** described in Remark 5. All the computations have been carried out on a PC Pentium 4 with the Matlab© LMI Toolbox.

Consider the multi-model linear time-varying system

$$x(t+1) = \sum_{i=1}^{2} p_i(t) A_i\, x(t) + \sum_{i=1}^{2} p_i(t) B_i\, u(t) \tag{33}$$

with

$$A_1 = \begin{bmatrix} 2 & -0.1 \\ 0.5 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0.1 \\ 2.5 & 1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ -0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.7 \\ 0.1 \end{bmatrix}, \tag{34}$$

and input saturation constraint $|u(t)| \leq 1,\ \forall t \geq 0$. The parameter vector $p(t)$ is assumed to be measurable at each time instant t. For all the simulations, we have considered the same conditions, i.e. the state and input weighting matrices $R_x = I, R_u = I$ in the quadratic performance index.

The off-line phase of the RHC algorithm has been initialized by choosing the following sequence of 8 states

$$X^{set} = \begin{bmatrix} -0.4 & -0.4102 & -04084 & -0.2144 & -0.2743 \\ 6 & 5.4903 & 4.9874 & 4.0754 & 3.7958 \end{bmatrix}$$
$$\begin{matrix} -0.2984 & -0.3294 & -0.0254 \\ 2.9065 & 2.1536 & 0.0522 \end{matrix} \Bigg].$$

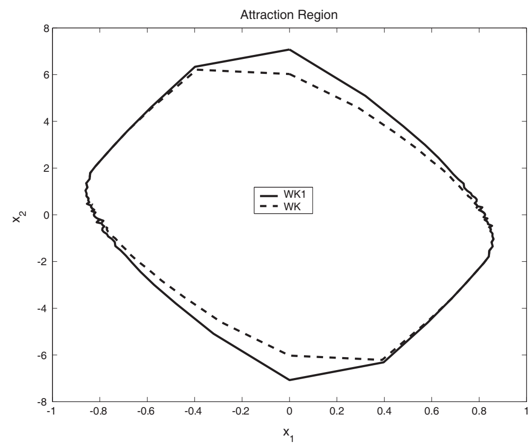

Fig. 2. State Attraction Region with input bound constraints - Under stabilizability conditions (31)-(32) (Continuous line), under stabilizability conditions (10) (Dashed line)

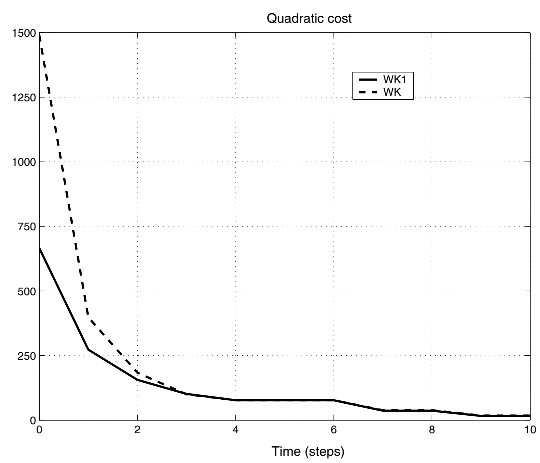

Fig. 3. Quadratic cost - Under stabilizability conditions (31)-(32) (Continuous line), under stabilizability conditions (10) (Dashed line)

Figure 2 reports for both versions of the RHC (**WK1** (continuous) and **WK** (dashed)). As it is evident, an enlarged region of feasible initial states results when our novel conditions are exploited (continuous line). Next, the on-line performance of the two versions of the algorithm has been compared by fixing the initial state to $x_0 = [-0.4\ \ 6]^T$ (admissible for both the strategies (see Fig. 2) and for a time-varying parameter $p(t) = [sin(t)\ 1 - sin(t)]$.

Again, the use of the proposed new conditions leads to a less conservative behavior. The improvement results by looking at the guaranteed costs in Fig. 3. Finally for the sake of completeness, the comparisons in terms of controlled input/regulated state time behavior are also depicted in Figs. 4-6.

VI. Conclusion

In this paper we have presented new stabilizability conditions for multi-model LPV discrete-time systems subject to input constraints.

A "refinement" of the classical conditions has been achieved

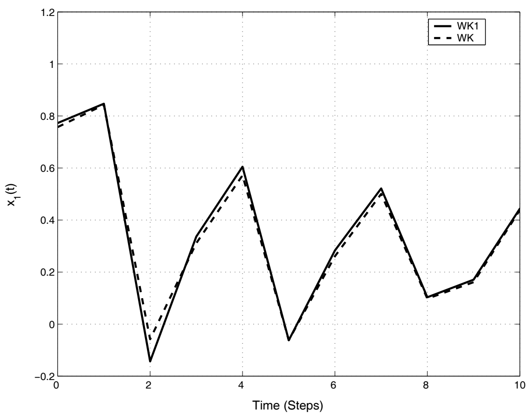

Fig. 4. State trajectory $x_1(t)$ - Under stabilizability conditions (31)-(32) (Continuous line), under stabilizability conditions (10) (Dashed line)

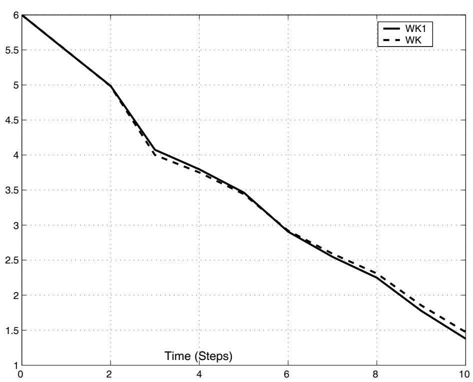

Fig. 5. State trajectory $x_2(t)$ - Under stabilizability conditions (31)-(32) (Continuous line), under stabilizability conditions (10) (Dashed line)

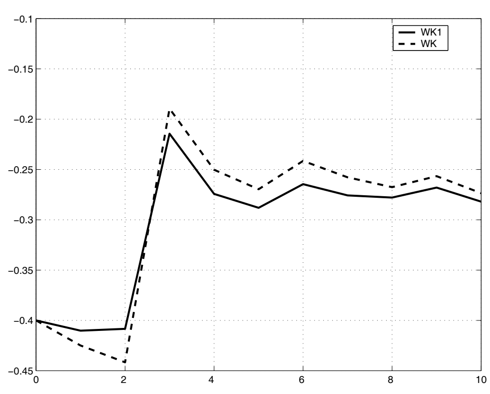

Fig. 6. Input signal - Under stabilizability conditions (31)-(32) (Continuous line), under stabilizability conditions (10) (Dashed line)

by resorting to a simple analysis on the structure of the scheduling parameters.

An application to a low demanding RHC problem has been provided and a numerical example presented in order to show the significant achieved improvements.

References

[1] P. Apkarian, P. Gahinet and G. Becker, Self-Scheduled H_∞ Control of Linear Parameter-Varying Systems: A Design Example, *Automatica*, Vol. 31, No. 9, pp. 1251-1261, 1995.

[2] P. Apkarian and H. D. Tuan Parameterized LMIs in Control Theory *SIAM Journal on Control and Optimization*, Vol. 38, No. 4, pp. 1241-1264, 2000.

[3] D. Angeli, A.Casavola and E.Mosca, "Predictive Control with Partial State Information", Proceedings of *European Control Conference 2001*, Porto, Portugal, 2001.

[4] A. Bemporad, M. Morari, V. Dua, and E.N. Pistikopoulos, "The explicit linear quadratic regulator for constrained systems," *Automatica*, Vol. 38, pp. 3-20, 2002.

[5] F. Blanchini and S. Miani, "Stabilization of LPV systems: state feedback, state estimation and duality", *SIAM Journal of Control and Optimization*, Vol. 42(1), pp. 76–97, 2003.

[6] F. Blanchini and S. Miani, "Stability results for continuous and discrete time linear parameter varying systems", *In Proc. 16th IFAC World Congress*, Prague, July 2005.

[7] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, *SIAM Studies in Applied Mathematics*, 15, SIAM, London.

[8] A. Casavola, D. Famularo and G. Franzè, Predictive control of constrained nonlinear system via LPV linear embeddings, *Int. J. Robust Nonlinear Control*, Vol. 13, 2003, pp 281-294.

[9] J. Daafouz and J. Bernussou, Parameter dependent Lyapunov functions for discrete time systems with time varying parametric uncertainties,*System & Control Letters*, Vol. 43, 2001, pp. 355-359.

[10] M.B. de Oliveira, J. Bernussou and J.C. Geromel, A new discrete-time robust stability condition,*System & Control Letters*, Vol. 37, 1999, pp. 261-265.

[11] J.C. Geromel, P.L.D. Peres and J. Bernussou, On a convex parameter space method for linear control design of uncertain sstems, *SIAM Journal of Control and Optimization*, Vol. 29, pp. 381-402, 1991.

[12] L. Imsland, N. Bar and B.A. Foss, "More efficient predictive control", *Automatica*, Vol. 41,pp. 1395-1403, 2005.

[13] T.A. Johansen, "Approximate explicit receding horizon control of constrained nonlinear systems, *Automatica*, Vol. 40, pp. 293-300, 2004.

[14] M.V. Kothare, V. Balakrishnan and M. Morari, Robust constrained model predictive control using linear matrix inequalities, *Automatica*, Vol. 32, 1996, pp. 1361-1379.

[15] B. Kouvaritakis, J.A. Rossiter and J. Schururmans, "Efficient robust predictive control", *IEEE Transactions on Automatic Control*, Vol. 45, pp. 1545-1549, 2000.

[16] G. Polya, Über positive Darstellung von Polynomen", Vterterhartschrift d. Naturforschenden Gessellschaft in Ziinch, Vol. 73, pp. 141-145, 1928. See, Collected Papers, Vol. 2, (MIT Press, Cambridge, 1974), pp. 309-313.

[17] C.W. Scherer and C.W.J. Hol, "Matri Sum of Squares of relaxation for robust semidefinite programs", *Mathematical programming*, Ser. B 107, pp. 189-211, 2006.

[18] J.G. VanAntwerp and R.D. Braatz, "Fast model predictive control of sheet and film processes", *IEEE Transactions on Control Systems Technology*, Vol. 8, pp. 408-417, 2000.

[19] Z. Wan and M.V. Kothare, "An efficient off-line formulation of robust model predictive control using linear matrix inequalities", *Automatica*, Vol. 39, pp. 837-846, 2003.

[20] H.O. Wang, K. Tanaka and F. Griffin, "An Approach to Fuzzy Control of Nonlinear Systems: Stability and Design Issues", *IEEE Transaction on Fuzzy Systems*, Vol. 4, No. 1, pp. 14-23, 1996.