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An improved predictive control strategy for polytopic LPV linear systems

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Abstract— **This paper presents a new dilated LMI approach to constrained MPC for LPV discrete-time systems. The main idea exploited in the proposed algorithm consists of using both the dilation argument and the direct measure of the LMI parameter at each time step. The effectiveness of the technique is compared to preexisting techniques by means of a numerical example.**

I. Introduction

Model Predictive Control (MPC) designates a family of optimization-based control strategies where, at each time instant, the input moves are computed according to future state predictions subject to a set of prescribed constraints.

The MPC literature is vast and covers a series of critical points and limitations amongst which the conservativeness of quadratic stabilization approach is especially of interest for uncertain systems [1]. A main open problem is in fact how to construct efficient robust MPC schemes under the latter paradigm or how to overcome it. Under the latter, existing solutions [1]-[5] suffer of conservativeness and exhibit adequate closed-loop performances only under restrictive operating conditions. Recently, starting from an idea by Bernussou et al. [6], less conservative quadratic stability conditions have been proposed in terms of dilation techniques (see [6]-[7]). The main idea of the dilation approach is the introduction of slack variables and a parameter dependent Lyapunov function whose effect is to relax the stability conditions at a price of introducing additional constraints. Nonetheless, extensive numerical comparisons revealed that the dilated approach is less conservative than the traditional one. Starting from these results, in [8] and [9] the dilation techniques have been proposed in a receding horizon fashion for an uncertain polytopic discrete-time plants subject to input, state and quadratic invariance constraints.

The contribution of this paper is to extend the proposed dilation Receding Horizon Control (RHC) strategy to the case of input saturated Linear Parameter Varying (LPV) systems expressed via polytopic or affine representations. Up to our's best acknowledge, this class of algorithms is missing in the MPC literature and potentially applicable to traditional LPV based receding horizon control algorithms proposed in literature (see [11]-[12] and references therein) to significantly reduce their intrinsic conservativeness.

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By assuming that the scheduling vector is measurable at each time instant, a parameter dependent scheduling control law is here proposed. Such a property is also exploited in the computations of the ellipsoidal quadratic invariance set (parameter dependent Lyapunov matrix). The closed-loop stability and feasibility properties of the proposed RHC scheme can be proved via standard arguments and are here summarized. A numerical example is considered and comparisons with traditional LPV algorithms and LPV dilation-based RHC schemes are finally reported.

The paper is organized as follows: the problem is stated in Section II where previous results are also described. In Section III, the dilated LPV RHC algorithm is presented, all required LMI conditions are derived and the feasibility and stability properties proved. A numerical experiment is reported in Section IV and some conclusions end the paper.

Notation

Given a matrix $T \in \mathcal{R}^{n\times n}$, we will denote with $T_{ii},\ i = 1,\dots,n$, the i-th diagonal entry.

II. Problem statement

Let us consider the following discrete-time uncertain linear system

$$\begin{cases} x(t+1) &= A(p(t))\,x(t) + B(p(t))\,u(t) \\ y(t) &= Cx(t) \end{cases} \tag{1}$$

with $x(t) \in \mathbf{R}^n$ denoting the state, $u(t) \in \mathbf{R}^m$ the input, $y(t) \in \mathbf{R}^p$ the output. The plant matrices have the following structure:

$$A(p) = \sum_{j=1}^{l} p_j A_j, \quad B(p) = \sum_{j=1}^{l} p_j B_j, \tag{2}$$

and are linearly dependent on a parameter vector $p = [p_1, p_2, \dots, p_l]^T \in \mathcal{P}$ with $\mathcal{P}$, the unit simplex

$$\mathcal{P} := \left\{ \sum_{j=1}^{l} p_j = 1,\ 0 \le p_j \le 1 \right\}. \tag{3}$$

The following additional property is satisfied by the family of plants (1)

- **LPV** - $p(t)$ is measurable at each time instant.

We will denote as $\mathcal{P}^r$ and $\mathcal{P}^*$ the sets of all possible $\mathcal{P}$-valued sequences of $r+1$ and arbitrary length, respectively,

$$\mathcal{P}^r := \left\{ \{p(t)\}_{t=0}^{r} : p(t) \in \mathcal{P} \right\}, \quad \mathcal{P}^* := \lim_{r\to\infty} \mathcal{P}^r. \tag{4}$$

Moreover, the plant input is subject to componentwise saturation input and 2-norm output constraints

$$|u_j(t)| \leq u_{j,\max} \; t \geq 0, j = 1, \ldots, m, \tag{5}$$

$$\|y(t)\|_2 \leq y_{max}, \; t \geq 0. \tag{6}$$

The aim is to find a state-feedback regulation strategy $u(t) = g(x(t))$ which asymptotically stabilizes (1)-(3) subject to (5) and (6).
Let us consider now, for a generic command sequence $u(\cdot)$, the following quadratic performance index

$$J(x, u(\cdot)) := \max_{p(\cdot)\in\mathcal{P}^*} \sum_{t=0}^{\infty} \{\|x(t)\|^2_{R_x} + \|u(t)\|^2_{R_u}\}, \tag{7}$$

with $R_u = R_u^T > 0$, $R_x = R_x^T > 0$ suitable input and state weights.
If the control strategy has a linear state-feedback form $u = Fx$, an upper bound to the cost (7) is given by (see [5] for details)

$$J(x(0), u(\cdot)) \leq x(0)^T \, P \, x(0) \tag{8}$$

with $P = P^T > 0$ satisfying the (robust) quadratic stabilization conditions [14] of the polytopic model (1)-(3)

$$A_{F,j}^T \, P \, A_{F,j} - P + F^T \, R_u \, F + R_x \leq 0, \quad j = 1, \ldots, l, \tag{9}$$

where $A_{F,j} := A_j + B_j \, F, \; R_x = C^T \, R_y \, C$.
More recently, for the sake of reducing conservativeness, (8)-(9) have been approached (see [6]) by considering a linearly parameter dependent Lyapunov function

$$P(p(t)) = \sum_{j=1}^{l} p_j(t) P_j, \tag{10}$$

instead of a fixed P in (9), where the matrix P is explicitly computed. Moreover, by introducing a matrix $G \in \mathbf{R}^{n\times n}$ acting like an additional degree of freedom and by means of congruence transformations, it is possible to rewrite (9) (see [8], [9]) as a set of less conservative "dilated" Matrix Inequalities

$$\begin{bmatrix} G + G^T - P_j^{-1} & * & * & * \\ A_{F,j}G & P_k^{-1} & * & * \\ R_x^{1/2}G & 0 & I & 0 \\ R_u^{1/2}FG & 0 & 0 & I \end{bmatrix} > 0 \tag{11}$$
$$j = 1, \ldots, l \quad k = 1, \ldots, l.$$

The price to be paid is an increased number of inequalities, growing quadratically with of the number of plant vertices. Given the p-parametric expression of P (10) and the dilated conditions (11), the upper bound (8) becomes

$$J(x(0), u(\cdot)) \leq x(0)^T \, P(p(0)) \, x(0), \tag{12}$$

where the **LPV** hypothesis and dilation technique are used. Moreover, the matrix P (or $P(p(0))$) is a shaping matrix for the following ellipsoidal set:

$$\mathcal{E}(P, \gamma) := \left\{x \in \mathbf{R}^n | \, x^T P x \leq \gamma, \;\; \gamma \geq 0\right\} \tag{13}$$

which can be proved to be a robust positively invariant region for the polytopic system (1)-(3) under the state-feedback F. In principle it is possible to reach the objective above described simply by computing an off-line couple (F, P) which guarantees feasibility for (9) (P fixed) or for (11) (P parameter depending and $p(0)$ measurable) under the input constraints (5) for a given initial state.
In order to reduce the implicit conservativeness of such a constrained quadratic stabilizing control law, a receding horizon paradigm will be here taken into consideration. One of the main consequences of the RHC approach is the use of a time-varying sequence of the pair $(F(t), P(t))$ instead of a fixed one.
By denoting as $\hat{s}(t+k|t)$ the k-steps ahead predictions of a generic system variable based on all information available at time t, here we will apply the following family of virtual commands:

$$\hat{u}(\cdot|t) = F(t)\hat{x}(t+k|t), \;\; k \geq 0 \tag{14}$$

where $F(t)$ is a sequence of stabilizing controllers compatible with the described constraints.
Therefore the problem we want to solve in a receding horizon fashion can be stated as:
Problem 1 - Determine at each time instant t, on the basis of the current state $x(t)$, a stabilizing pair $(P(t), F(t))$ which minimizes an upper bound to the cost index $J(x(t), u(\cdot))$ such that the following conditions are satisfied from t onward:

- Input constraints: $|u_j(t)| \leq u_{j,\max}, \; t \geq 0, j = 1, \ldots, m$;
- Output constraints: $\|y(t)\|_2 \leq y_{max}, \; t \geq 0$;
- Invariance condition: $x(t) \in \mathcal{E}(P(t), \gamma(t))$.

A robust MPC paradigm (see Cuzzola et al. [8]) partially revised in [9], which exploits the dilation techniques and makes explicit use of the parameter dependent Lyapunov function (10), has been recently proposed to solve this problem.

Let $x(t)$ be the measured state, $P(t) = \sum_{j=1}^{l} p_j(t) P_j$ a parameter dependent Lyapunov function and $\hat{u}(t+i|t) = F(t)\hat{x}(t+i|t), \; i \geq 0$ the adopted control strategy. The following optimization solves **Problem 1** at each time step t :

$$\min_{Y,G,Q_j} \;\; \gamma \tag{15}$$

subject to

$$\begin{bmatrix} G + G^T - Q_j & * & * & * \\ A_j + B_jY & Q_k & * & * \\ R_x^{1/2}G & 0 & \gamma I & 0 \\ R_u^{1/2}Y & 0 & 0 & \gamma I \end{bmatrix} > 0 \tag{16}$$
$$k = 1, \ldots, l \quad j = 1, \ldots, l,$$

$$\begin{bmatrix} 1 & x(t)^T \\ x(t) & Q_j \end{bmatrix} \geq 0, \;\; j = 1, \ldots, l, \tag{17}$$

$$\begin{aligned}&\left[\begin{array}{cc} X & Y \\ Y^T & G+G^T-Q_j \end{array}\right] \geq 0, \quad j=1,\ldots,l, \\ &X_{kk} \leq u_{k,\max}^2 \quad k=1,\ldots,m,\end{aligned} \tag{18}$$

$$\left[\begin{array}{cc} G+G^T-Q_j & * \\ C(A_jG+B_jY) & y_{max}^2 I \end{array}\right] \geq 0, \quad j=1,\ldots,l, \tag{19}$$

where the usual variable changes [14] have been used

$$\begin{array}{rcl} P_j &=& \gamma Q_j^{-1}, \ j=1,\ldots,l, \\ F &=& Y G^{-1}. \end{array} \tag{20}$$

Note that (15)-(19) do not explicitly take into account the **LPV** hypothesis. Therefore, it can be regarded as a robust control algorithm effective for uncertain polytopic plants.

III. Dilated MPC Scheme for LPV Systems and Main Results

In this section we will give an LMI-based solution to **Problem 1** by exploiting the **LPV** hypothesis. The starting point is the LMI procedure (15)-(19) described in the previous section. Let us state our main result:

Proposition 1 *Consider the uncertain system (1) under the* **LPV** *hypothesis and let $x(t)$ be the measured state. Then, at each time step t the following optimization*

$$\min_{Y,G,Q_j} \ \gamma \tag{21}$$

subject to

$$\left[\begin{array}{cccc} G+G^T-\left(\frac{Q_i+Q_j}{2}\right) & * & * & * \\ \left(\frac{A_iG+A_jG+B_iY_j+B_jY_i}{2}\right) & Q_k & * & * \\ R_x^{1/2}G & 0 & \gamma I & * \\ R_u^{1/2}\left(\frac{Y_i+Y_j}{2}\right) & 0 & 0 & \gamma I \end{array}\right] > 0 \tag{22}$$
$$k=1,\ldots,l \quad i=1,\ldots,l \quad j=i,\ldots,l$$

$$\left[\begin{array}{cc} 1 & x^T(t) \\ x(t) & Q_j \end{array}\right] > 0, \quad j=1,\ldots,l \tag{23}$$

$$\begin{aligned}&\left[\begin{array}{cc} X & Y_j \\ Y_j^T & G^T+G-Q_j \end{array}\right] \geq 0, \quad j=1,\ldots,l \\ &X_{kk} \leq u_{k,\max}^2 \quad k=1,\ldots,m,\end{aligned} \tag{24}$$

$$\left[\begin{array}{cc} G+G^T-\frac{Q_i+Q_j}{2} & * \\ C\left(\frac{A_iG+A_jG+B_iY_j+B_jY_i}{2}\right) & y_{max}^2 I \end{array}\right] \geq 0, \tag{25}$$
$$i=1,\ldots,l, \ j=i,\ldots,l,$$

solves **Problem 1**, *with $F(t)$ defined as follows:*

$$F(t) := \sum_{j=1}^{l} p_j(t) F_j. \tag{26}$$

where $F_j = Y_j G^{-1}, \ j=1,\ldots,l$, and $P(t)$:

$$P(t) := \sum_{j=1}^{l} p_j(t) P_j, \tag{27}$$

with $P_j = \gamma\, Q_j^{-1}$.

Proof - Let us first consider the guaranteed cost conditions (22)-(23). At each time step t we want to determine sufficient conditions under which a state feedback control law $\hat{u}(t+\tau|t) = F(t)\hat{x}(t+\tau|t)$, $\tau \geq 0$ robustly stabilizes (1) and minimizes the cost function $J(x(t), u(\cdot|t))$. To this end, we define the quadratic function (see [8], [6])

$$V(\tau,t) = \hat{x}(t+\tau|t)^T\, P(\tau,t)\, \hat{x}(t+\tau|t), \ \ \forall \tau, t \geq 0, \tag{28}$$

with

$$P(\tau,t) := \sum_{j=1}^{l} p_j(t+\tau)\, P_j, \ \ \tau \geq 0. \tag{29}$$

In order to satisfy the upper bound (12), the 1-step ahead difference w.r.t. τ of (28) must satisfy the inequality

$$\begin{aligned}&V(\tau+1,t) - V(\tau,t) \\ &\leq -\left[\hat{x}^T(t+\tau|t) R_x \hat{x}(t+\tau|t) + u^T(t+\tau|t) R_u u(t+\tau|t)\right] \\ &\forall p(t+\tau) \in \mathcal{P}^*, \ \ \tau \geq 0.\end{aligned} \tag{30}$$

The $\tau+1-$steps ahead predictions under the control strategy (14) is given by $\hat{x}(t+\tau+1|t) = A_F(\tau,t)\, \hat{x}(t+\tau|t)$ and as a consequence (30) becomes

$$\begin{aligned}&\hat{x}^T(t+\tau|t)\left[A_F^T(\tau,t)\, P(\tau+1,t)\, A_F(\tau,t) - P(\tau,t)\right. \\ &\left. +F(t)^T R_u F(t) + R_x\right] \hat{x}(t+\tau|t) \leq 0\end{aligned} \tag{31}$$

with

$$\begin{aligned} A_F(\tau,t) &:= \sum_{j=1}^{l} p_j(t+\tau)\, \left(A_j + B_j\, F(t)\right), \quad (32) \\ P(\tau+1,t) &:= \sum_{k=1}^{l} p_k^+(t+\tau)\, P_k, \ \ \tau \geq 0, \quad (33)\end{aligned}$$

where $p_k^+(t+\tau)$ denotes the one-step ahead variation of the scheduling parameter $p(\cdot)$. By means of Schur complements, (31) is satisfied if the following matrix inequality holds

$$\left[\begin{array}{cccc} P(\tau,t) & * & * & * \\ P(\tau+1,t)A_F(\tau,t) & P(\tau+1,t) & * & * \\ R_x^{1/2} & 0 & I & * \\ R_u^{1/2}\ F(t) & 0 & 0 & I \end{array}\right] > 0, \ \forall t, \tau \geq 0. \tag{34}$$

Inequality (34) holds true if

$$\left[\begin{array}{cccc} P(\tau,t) & * & * & * \\ P_k A_F(\tau,t) & P_k & * & * \\ R_x^{1/2} & 0 & I & * \\ R_u^{1/2}\ F(t) & 0 & 0 & I \end{array}\right] > 0, \quad k=1,\ldots,l \tag{35}$$

is satisfied over the vertices of $P(\tau+1,t)$ (eq. (33)). Then, by denoting $Q_k = \gamma P_k^{-1}$ and applying the congruence transformation diag$(\gamma^{-1/2}G^T, \gamma^{-1/2}Q_k, I, I)$ to (35), one obtains that matrix inequality (34) is satisfied if

$$\left[\begin{array}{cccc} \gamma^{-1}G^T P\,(\tau,t)\, G & * & * & * \\ A_F(\tau,t)\, G & Q_k & * & * \\ \gamma^{-1/2}R_x^{1/2}G & 0 & I & * \\ \gamma^{-1/2}R_u^{1/2}F\,(t)\, G & 0 & 0 & I \end{array}\right] > 0, \quad k=1,\ldots,l. \tag{36}$$

By following the same lines as in [12], $A_F(\tau,t)$, $F(t)$ and $P(\tau,t)$ can be proved to be equal to

$$A_F(\tau,t) = \sum_{i=1}^{l}\sum_{j\geq i}^{l} 2\,p_i(t+\tau)\,p_j(t+\tau)\,\left(\frac{A_i+A_j+B_jF_i+B_iF_j}{2}\right) \quad (37)$$

$$F(t) = \sum_{i=1}^{l}\sum_{j\geq i}^{l} 2\,p_i(t)\,p_j(t)\,\left(\frac{F_i+F_j}{2}\right) \quad (38)$$

$$P(\tau,t) = \sum_{i=1}^{l}\sum_{j\geq i}^{l} 2\,p_i(t+\tau)\,p_j(t+\tau)\,\left(\frac{P_i+P_j}{2}\right) \quad (39)$$

and a sufficient condition, which guarantees the validity of (36), is given by

$$\begin{bmatrix} \gamma^{-1}\left(\frac{G^TP_iG+G^TP_jG}{2}\right) & * & * & * \\ \left(\frac{A_i+A_j+B_iF_j+B_jF_i}{2}\right)G & Q_k & * & * \\ \gamma^{-1/2}R_x^{1/2}G & 0 & I & * \\ \gamma^{-1/2}R_u^{1/2}\left(\frac{F_i+F_j}{2}\right)G & 0 & 0 & I \end{bmatrix} > 0 \quad (40)$$
$$i=1,\ldots,l \quad j=i,\ldots,l \quad k=1,\ldots,l.$$

By applying the *dilation trick* (see [7]), namely

$$G^T\,S^{-1}G \geq G+G^T-S$$

to the $(1,1)$ block of (40) the condition (22) results via the standard variable change $F_i = Y_i\,G^{-1}$. The invariance condition (23) can be trivially proved by the same arguments used in [5]. Let us now consider the input constraint condition

$$|\hat{u}_j(t+\tau|t)| \leq u_{j,\max},\ \tau\geq 0,\ \ j=1,\ldots,m. \quad (41)$$

Using a series of well-known technicalities (see [5]), (41) is satisfied if

$$\max_{\tau\geq 0}|\hat{u}_j\,(t+\tau|t)|^2 \leq \max_{\tau\geq 0}\left|(F(t)\,\hat{x}\,(t+\tau|t))_j\right|^2 \leq \max_{z\in\mathcal{E}}\left|(F(t)\,z)_j\right|^2. \quad (42)$$

Introducing $W(\tau,t) = \gamma\,P^{-1}(\tau,t)$, one has

$$\max_{z\in\mathcal{E}}\left|(F(t)\,z)_j\right|^2 = \max_{z\in\mathcal{E}}\left|\left(F(t)W^{1/2}(\tau,t)W^{-1/2}(\tau,t)z\right)_j\right|^2,$$

which implies

$$\max_{z\in\mathcal{E}}\left|\left(F(t)\,W^{-1/2}(\tau,t)\,W^{-1/2}(\tau,t)\,z\right)_j\right|^2$$

$$\leq \left\|\left(F(t)\,W(\tau,t)^{1/2}\right)_j\right\|_2^2$$

$$= \left(F(t)\,W(\tau,t)\,F^T(t)\right)_{jj} \leq u_{j,\max}^2.$$

By imposing the existence of a matrix $X = X^T > 0$ such that

$$X_{jj} \leq u_{j,\max}^2\,, \quad (43)$$

we obtain that (41) is implied by

$$\begin{bmatrix} X & F(t) \\ F^T(t) & W^{-1}(t,\tau) \end{bmatrix} \geq 0. \quad (44)$$

Finally, by applying the congruence transformation $\mathrm{diag}\,(I,G))$ to (44) and recalling the expressions of $F(t)$ and $P(\tau,t)$ (26)-(29), (44) becomes

$$\begin{bmatrix} X & Y_j \\ Y_j^T & G^TQ_j^{-1}G \end{bmatrix} \geq 0,\ j=1,\ldots,l. \quad (45)$$

and easily dilated as in (24).

The proof of (25) can be easily derived by similar arguments as proposed in [5], [6] and [12].

□

All above developments allows one to write down a computable MPC scheme, hereafter denoted as *DL-LPV*, which consists of the following algorithm.

DL-LPV

1.1 `At each time instant` $t \geq 0$`, given` $x(t|t)$ `and` $p(t)$`, solve`

$$[Y_{j,\text{opt}}(t),\,Q_{j,\text{opt}}(t),\,G_{\text{opt}}(t),\,\gamma_{\text{opt}}(t)] \triangleq \arg\min_{Y_j,\,Q_j,\,G,\,\gamma}\gamma \quad (46)$$

`subject to the constraints (22), (23), (24), (25).`
`Compute`

$$F(t) = \sum_{j=1}^{l} p_j(t)\,Y_{j,\text{opt}}\,G_{\text{opt}}^{-1};$$

1.2 `feed the plant with` $u(t) = F(t)x(t|t)$`;`
1.3 $t = t+1$ `and go to step 1.1`

Theorem 1 *Let the DL-LPV scheme has a solution at time $t = 0$ (point 1.1). Then, it has solution at each future time instants t, satisfies the input constraints and yields an asymptotically stable closed-loop system.*

Proof - Follows same arguments from [5]-[9]. □

Remark 1 - It is worth to note that the standard quadratic stability conditions under the **LPV** hypothesis, proved in [12], can be recovered by imposing $G = Q_j,\ \forall j = 1,\ldots,l$ in (22). This means that the results under the standard quadratic approach are more restrictive and, as a consequence, the scheme here proposed is not more conservative than the RHC scheme developed in [12]. □

IV. Numerical Experiment

The aim of this numerical section is to test the effectiveness of the proposed RHC strategy. To this end, the scheme will be compared in terms of control performances and computational complexity with the algorithm given in [12] (NDL-LPV), by using $N = 0$ free control moves, and with the *dilated*-based MPC scheme of [8] adapted to the LPV framework. All the computations have been carried out on a PC Pentium 4-based using the `YALMIP` Toolbox [15]. Consider the polytopic uncertain system

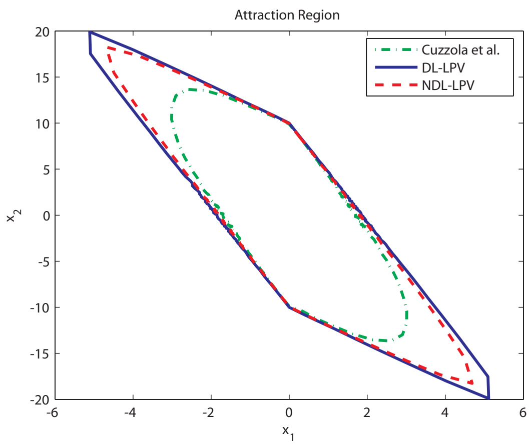

Fig. 1. State Attraction Region with input bound constraints - DL-LPV (Continuous line), NDL-LPV (Dashed line), Cuzzola et al. (Point-Dashed line)

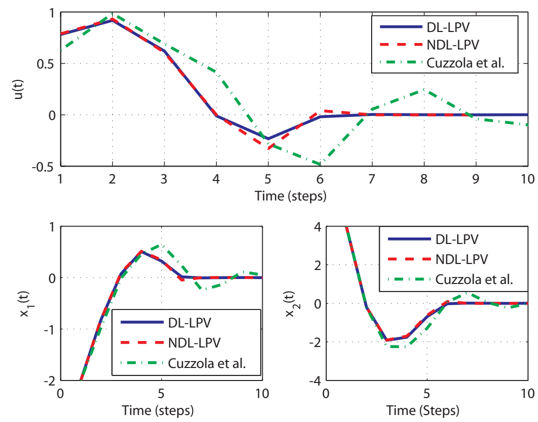

Fig. 2. Regulated state trajectories and control input $x(0) = [-2 \;\; 4]^T$, - DL-MPC (Continuous line), NDL-LPV (Dashed line), Cuzzola et al. (Point-Dashed line)

$$
\begin{aligned}
x(t+1) &= \sum_{i=1}^{2} p_i(t) A_i \, x(t) + \sum_{i=1}^{2} p_i(t) B_i \, u(t) \\
y(t) &= Cx(t)
\end{aligned} \qquad (47)
$$

where the system matrix vertices are

$$
A_1 = \begin{bmatrix} 1 & 0.1 \\ 0.5 & 1 \end{bmatrix}
$$

$$
A_2 = \begin{bmatrix} 1 & 0.1 \\ 2.5 & 1 \end{bmatrix} \qquad (48)
$$

$$
B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
$$

and the input $u(t)$ is subject to the following saturation constraints

$$
|u(t)| \leq 1, \;\; \forall t \geq 0. \qquad (49)
$$

The parameter vector $p(t)$ is assumed to be measurable at each time instant t. For all the three schemes, we have considered the same simulation conditions, i.e. the state and input weighting matrices $R_x = I, R_u = I$ in the quadratic performance index (7).

The first figure reports the attraction basins for the three algorithms. As it clearly results in Fig. 1, the DL-LPV scheme (continuous line) shows an enlarged region of feasible initial states, admitting a solution to the constrained optimization problem. Further, it is worth to note that the attraction domain of the NDL-LPV scheme encloses that of [8].

Moreover, we have compared the three schemes in terms the regulated responses by fixing the initial state to $x_0 = [-2 \;\; 4]^T$ (admissible for all the three strategies, see Fig. 1). In Fig. 2 are depicted the comparisons in terms of control performances. As expected, the DL-LPV scheme shows better regulated responses, especially when compared with the RHC algorithm of [8]. It is worth noting that DL-LPV and NDL-LPV seem to perform almost in the same manner. On the other hand, Fig. 3 shows that the proposed RHC strategy works better w.r.t. the other two schemes.

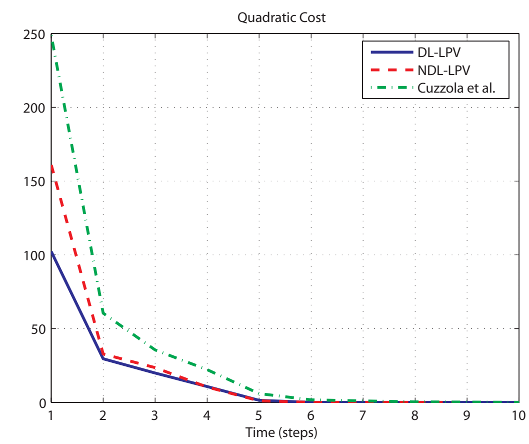

Fig. 3. Quadratic cost - DL-LPV (Continuous line), NDL-LPV (Dashed line), Cuzzola et al. (Point-Dashed line)

Finally, in Table I, the computational burdens have been computed in terms of average CPU time. Note that the price paid by DL-LPV to improve the control performances is slightly greater w.r.t. the Cuzzola *et al.* and NDL-LPV paradigms.

V. Conclusions

A novel moving horizon control strategy for input-saturated Polytopic Linear Parameter Varying (LPV) systems has been described. The control strategy is an extension to the LPV case of a receding horizon control scheme recently proposed in literature, which takes advantage of the so-called dilation techniques that are less conservative w.r.t. traditional quadratic stability approaches. By exploiting the property that the parameter vector is available at each time instant, a scheduling control strategy has been derived

TABLE I

ON-LINE NUMERICAL BURDENS: FLOPS PER STEP (AVERAGE CPU TIME-SECONDS PER STEP)

Algorithm	CPU time (average)
Cuzzola *et al.* [8]	1.3951
NDL-LPV	1.3683
DL-LPV	1.7121

by means of an ellipsoidal invariant set which depends on a parameter-dependent time-varying Lyapunov matrix. A numerical experiment has shown the benefits of the proposed MPC strategy.

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